

RINGS OF SATURN

FOR THREE RETUNED, COMPUTER-DRIVEN PIANOS

BY KYLE GANN
2016-17

Technical Specifications

The 33-pitch tuning of the three pianos (the same in every octave) is as follows, given first in the number of cents above E-flat, and then as ratios to the nearest E-flat 1/1 below:

Piano	1	2	3			
D	1088	15/8	977	225/128	1044	117/64
Db	969	7/4	938	55/32	906	27/16
C	857	105/64	773	25/16	840	13/8
B	738	49/32	755	99/64	729	195/128
Bb	702	3/2	590	45/32	609	91/64
A	551	11/8	551	11/8	481	169/128
Ab	471	21/16	440	165/128	408	81/64
G	386	5/4	320	77/64	342	39/32
Gb	204	9/8	275	75/64	275	75/64
F	155	35/32	192	143/128	192	143/128
E	92	135/128	53	33/32	27	65/64
Eb	0	1/1	1103	121/64	1173	63/32

Note that no string needs to be raised higher than its natural tuning except for the B-flat on piano 1, which is 2¢ sharp (or if one prefers, 2¢ could be subtracted from all quantities).

For electronic realization of the piece, it can prove helpful to reconfigure the tuning as a reference pitch in cycles per second for each piano, and ratios derived from that standard:

Tuning pitch: 38.891 cps	36.7641 cps	38.2833 cps	
D	15/8	225/121	13/7
Db	7/4	20/11	12/7
C	105/64	200/121	104/63
B	49/32	18/11	65/42
Bb	3/2	180/121	13/9
A	11/8	16/11	169/126
Ab	21/16	15/11	9/7
G	5/4	14/11	26/21
F#	9/8	150/121	25/21
F	35/32	13/11	143/126
E	135/128	12/11	65/63
Eb	1/1	1/1	1/1

In the configuration of certain tuning softwares, the following sequences might facilitate getting the required tuning:

Piano 1:

38.891 = Eb0

1/1, 135/128, 35/32, 9/8, 5/4, 21/16, 11/8, 3/2, 49/32, 105/64, 7/4, 15/8

Piano 2:

36.7641485 = Eb0

1/1, 12/11, 13/11, 150/121, 14/11, 15/11, 16/11, 180/121, 18/11, 200/121, 20/11, 225/121

Piano 3:

38.283333 = Eb0

1/1, 65/63, 143/126, 25/21, 26/21, 9/7, 169/126, 13/9, 65/42, 104/63, 12/7, 13/7

For purposes of analysis, the entire scale (which I refer to as my 8x8 scale) is given below, grouping its pitches into eight harmonic series' on the 1st, 3rd, 5th, 7th, 9th, 11th, 13th, and 15th harmonics of E-flat, and naming each pitch in a typographical equivalent of Ben Johnston's just-intonation notation:

Pitch name	Ratio	Cents	1/1	3/2	5/4	7/4	9/8	11/8	13/8	15/8
Db ^{^^} -	121/64	1103						11		
D	15/8	1088	15	5	3				1	
Db13	117/64	1044					13		9	
C#+	225/128	977								15
<u>Db7</u>	<u>7/4</u>	<u>969</u>	<u>7</u>			<u>1</u>				
C^	55/32	938			11			5		
C+	27/16	906		9			3			
C7+	105/64	857				15				7
Cb13	13/8	840	13					1		
<u>B</u>	<u>25/16</u>	<u>773</u>			<u>5</u>					
Bb^	99/64	755					11	9		
Cb77+	49/32	738				7				
Bb13	195/128	729						15	13	
Bb	3/2	702	3	1						
<u>Bbb713</u>	<u>91/64</u>	<u>609</u>			<u>13</u>			7		
A+	45/32	590		15	9		5			3
Ab^	11/8	551	11				1			
Abb1313	169/128	481						13		
Ab7+	21/16	471		7		3				
G^	165/128	440					15		11	
G+	81/64	408					9			
G	5/4	386	5		1					
Gb13	39/32	342		13					3	
Gb7^	77/64	320				11		7		
F#+	75/64	275			15					5
F+	9/8	204	9	3			1			
Fb13^	143/128	192						13	11	
F7+	35/32	155			7	5				
E+	135/128	92					15			9
Eb^	33/32	53		11				3		
Eb13	65/64	27			13				5	
Eb	1/1	0		1						
Eb7+	63/32	1173				9	7			

In Johnston's notation, + raises a pitch by 81/80, # raises it by 25/24, b lowers it by 24/25, 7 lowers it by 35/36, ^ raises it by 33/32, 13 raises it by 65/64, and F-A-C, C-E-G, and G-B-D are all perfectly tuned 4:5:6 major triads.

Rings of Saturn

Kyle Gann
2016-17

$\text{♩} = 76$

Piano 2

$\text{♩} = 76$

Piano 2

mp

p

p



7

Pno2

7

Pno2

p

3

2

II

Pno2

3

3

≡

15

Pno1

Pno2

p

p

p

3

3

7:6

Rings of Saturn

19

Pno1

Pno2

3

23

Pno1

Pno2

Pno3

p

Rings of Saturn

27

Pno1

Pno2

Pno3

5

Rings of Saturn

6

31

Pno1

Pno2

Pno3

5:4

5

Rings of Saturn

34

Pno1

Pno2

Pno3

7

Rings of Saturn

37

Pno1

b-flat

II

II

b-flat.

Pno2

II

II

II

II

σ

σ..

σ.

Pno3

II

II

II

5

II

II

Rings of Saturn

41

Pno1

Pno2

Pno3

9

Rings of Saturn

10

44

Pno1

Pno2

Pno3

48

3

Pno2

Rings of Saturn

51

11

The musical score consists of two systems of piano parts. The top system, labeled 'Pno1', has two staves: treble and bass. The bottom system, labeled 'Pno2', also has two staves: treble and bass. Measure 51 begins with both pianos silent. Pno1 starts with a single eighth note in the bass staff at the end of measure 51. Pno2 enters with a sixteenth-note pattern in the treble staff at the beginning of measure 52. Measure 11 follows, with Pno1 playing a rhythmic pattern of eighth and sixteenth notes in the bass staff, and Pno2 continuing its sixteenth-note pattern in the treble staff.

Rings of Saturn

55

Pno1

Pno2

59

Pno1

13

13

13

Pno2

13

13

13

Rings of Saturn

62

Pno1

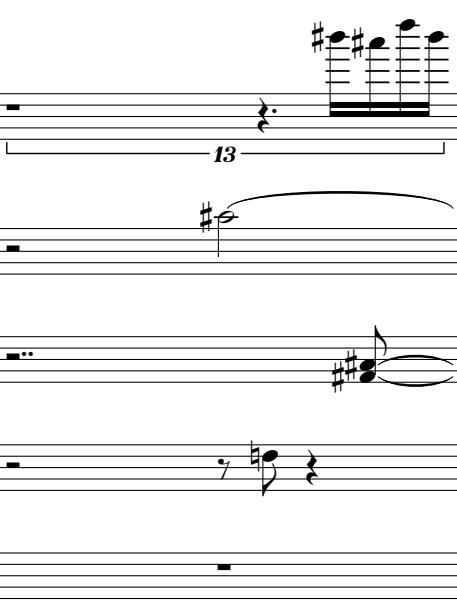
Pno2

65 
13

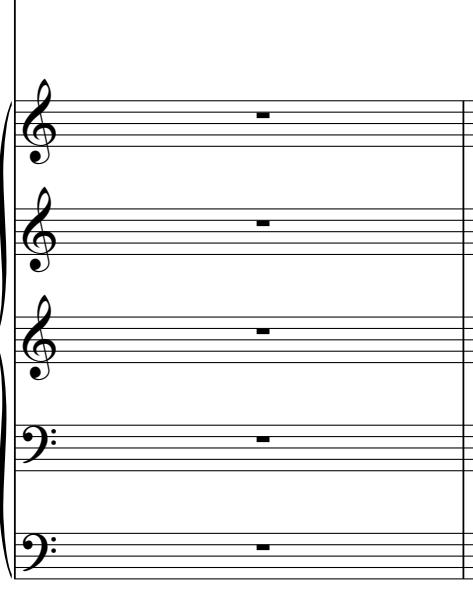
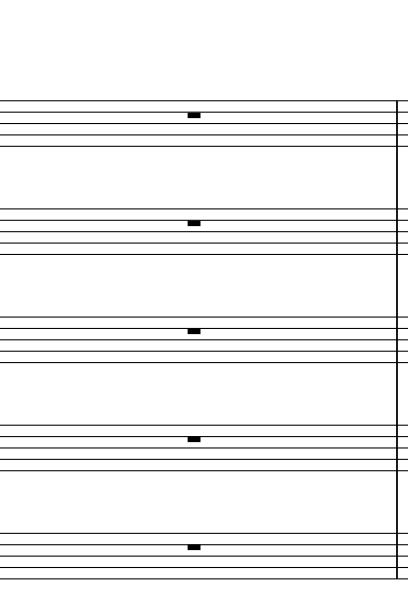
13

13

Pno1

Pno2

Pno3




Rings of Saturn

16

68

This musical score page contains three systems of piano music, labeled Pno1, Pno2, and Pno3, arranged vertically. Each system has three staves: treble, middle, and bass. The music is divided into measures by vertical bar lines. Measures are grouped into sections by horizontal bracketing. Measure numbers are placed above the first measure of each section.

- Pno1:** The first section (measures 1-13) features eighth-note patterns in the treble and middle staves, with rests in the bass. The second section (measures 14-26) shows more complex patterns, including sixteenth-note figures and sustained notes. The third section (measures 27-40) includes sustained notes and eighth-note chords.
- Pno2:** The first section (measures 1-13) consists of eighth-note patterns in the treble and middle staves, with rests in the bass. The second section (measures 14-26) features eighth-note chords in the treble and middle staves, with rests in the bass. The third section (measures 27-40) includes sustained notes and eighth-note chords.
- Pno3:** The first section (measures 1-13) has rests in all three staves. The second section (measures 14-26) includes eighth-note patterns in the treble and middle staves, with rests in the bass. The third section (measures 27-40) features eighth-note patterns in the treble and middle staves, with rests in the bass.

Musical score for three pianos (Pno1, Pno2, Pno3) in 13-measure sections. The score is divided into three systems by vertical bar lines.

- Pno1:** The first system shows Pno1 playing eighth-note chords in measures 1-2, followed by eighth-note patterns in measures 3-4, and sixteenth-note patterns in measures 5-6. Measures 7-8 show eighth-note chords, and measures 9-10 show eighth-note patterns. Measure 11 is a rest, and measure 12 shows eighth-note chords again. The third system begins with eighth-note chords in measures 13-14, followed by eighth-note patterns in measures 15-16, and sixteenth-note patterns in measures 17-18.
- Pno2:** The first system shows Pno2 playing eighth-note chords in measures 1-2, followed by eighth-note patterns in measures 3-4, and sixteenth-note patterns in measures 5-6. Measures 7-8 show eighth-note chords, and measures 9-10 show eighth-note patterns. Measure 11 is a rest, and measure 12 shows eighth-note chords again. The third system begins with eighth-note chords in measures 13-14, followed by eighth-note patterns in measures 15-16, and sixteenth-note patterns in measures 17-18.
- Pno3:** The first system shows Pno3 playing eighth-note chords in measures 1-2, followed by eighth-note patterns in measures 3-4, and sixteenth-note patterns in measures 5-6. Measures 7-8 show eighth-note chords, and measures 9-10 show eighth-note patterns. Measure 11 is a rest, and measure 12 shows eighth-note chords again. The third system begins with eighth-note chords in measures 13-14, followed by eighth-note patterns in measures 15-16, and sixteenth-note patterns in measures 17-18.

Rings of Saturn

7/4

Pno1

Pno2

Pno3

Rings of Saturn

77

Pno1

Pno2

Pno3

Rings of Saturn

80

Pno1

Pno2

Pno3

Rings of Saturn

21

83

Pno1

Pno2

Pno3

Rings of Saturn

Musical score for piano part Pno1, measures 87-89. The score consists of two systems of four staves each. Measure 87 starts with a rest followed by a dynamic *pp*. Measure 88 begins with a bass note, followed by a dynamic *pp*. Measure 89 concludes with a dynamic *p*.

Musical score for piano part Pno2, measures 87-89. The score consists of two systems of four staves each. Measure 87 features a bass line with a dynamic *pp*. Measure 88 includes a treble line with a dynamic *pp*. Measure 89 ends with a dynamic *p*.

Musical score for piano part Pno3, measures 87-89. The score consists of two systems of four staves each. Measure 87 contains a bass line. Measure 88 includes a treble line with a dynamic *pp*. Measure 89 ends with a dynamic *p*.

Rings of Saturn

90

Pno1

Pno2

Pno3

Rings of Saturn

93

Pno1

Pno2

Pno3

Rings of Saturn

97

Pno1

25

Pno2

p

Pno3

p

p

Rings of Saturn

101

Pno1

Pno2

Pno3

105

Pno1

Pno2

Pno3

Rings of Saturn

109

Rings of Saturn

113

Pno1

29

Pno2

Pno3

Rings of Saturn

117

Pno1

Pno2

Pno3

mp

Rings of Saturn

120

Pno1

p

pp

pp

pp

p

pp

p

Pno2

pp

pp

pp

Pno3

pp

p

p

p

Rings of Saturn

124

Pno1

Pno2

Pno3

Rings of Saturn

Musical score for Pno1 (Piano 1) at measure 128. The score consists of four staves. The top staff is treble clef, the second is bass clef, and the third and fourth are bass clef. The music features various note heads, stems, and rests. Measure 128 starts with a rest followed by a series of eighth and sixteenth notes. Measure 129 begins with a bass note, followed by a series of eighth and sixteenth notes. Measure 130 starts with a bass note, followed by a series of eighth and sixteenth notes. Measure 131 starts with a bass note, followed by a series of eighth and sixteenth notes.

Musical score for Pno2 (Piano 2) at measure 132. The score consists of four staves. The top staff is treble clef, the second is bass clef, and the third and fourth are bass clef. The music features various note heads, stems, and rests. Measure 132 starts with a rest followed by a series of eighth and sixteenth notes. Measure 133 begins with a bass note, followed by a series of eighth and sixteenth notes. Measure 134 starts with a bass note, followed by a series of eighth and sixteenth notes. Measure 135 starts with a bass note, followed by a series of eighth and sixteenth notes.

Musical score for Pno3 (Piano 3) at measure 136. The score consists of four staves. The top staff is treble clef, the second is bass clef, and the third and fourth are bass clef. The music features various note heads, stems, and rests. Measure 136 starts with a rest followed by a series of eighth and sixteenth notes. Measure 137 begins with a bass note, followed by a series of eighth and sixteenth notes. Measure 138 starts with a bass note, followed by a series of eighth and sixteenth notes. Measure 139 starts with a bass note, followed by a series of eighth and sixteenth notes.

Rings of Saturn

132

The musical score consists of three systems, each for a piano (Pno1, Pno2, Pno3). The score is in common time. Measure 34 begins with a rest followed by a dynamic of *p*. The first system (Pno1) features a treble clef, a bass clef, and a bass clef. It contains eighth-note patterns and rests. The second system (Pno2) also has a treble clef, a bass clef, and a bass clef, with similar eighth-note patterns and rests. The third system (Pno3) has a treble clef, a bass clef, and a bass clef, with eighth-note patterns and rests. Measures 35 and 36 show further continuation of these patterns. Measure 37 concludes with a dynamic of *pp* and the text "Rings of Saturn".

Pno1

Pno2

Pno3

Rings of Saturn

135

Pno1

Pno2

Pno3

Rings of Saturn

138

Pno1

Pno2

Pno3

Rings of Saturn

141

Pno1

Pno2

Pno3

Rings of Saturn

145

Pno1

Pno2

Pno3

Rings of Saturn

149

Pno1

39

Pno2

p

Pno3

p

Rings of Saturn

152

Pno1

Pno2

Pno3

155

 Pno1

 Pno2

 Pno3

Rings of Saturn

158

Pno1

Pno2

Pno3

mp

Rings of Saturn

161

 Pno1

 Pno2

 Pno3

Rings of Saturn

164

Pno1

19

19

Pno2

19

19

mf

Pno3

19

19

Rings of Saturn

Musical score for three pianos (Pno1, Pno2, Pno3) at measure 166. The score consists of three systems, one for each piano.

Pno1: The first system shows Pno1's part. It features a treble clef, a bass clef, and a bass clef. The music includes sixteenth-note patterns, eighth-note pairs, and various rests. Measure 166 starts with a treble clef, followed by a bass clef, and then a bass clef. Measure 167 begins with a bass clef. The dynamics include a dynamic marking *p* at the start of the second measure. Measure 168 starts with a bass clef. Measures 166-168 are grouped by a bracket labeled "19". Measures 169-170 are also grouped by a bracket labeled "19".

Pno2: The second system shows Pno2's part. It features a treble clef, a bass clef, and a bass clef. The music includes eighth-note pairs and rests. Measure 166 starts with a treble clef, followed by a bass clef, and then a bass clef. Measure 167 begins with a bass clef. Measure 168 starts with a bass clef. Measures 166-168 are grouped by a bracket labeled "19". Measures 169-170 are grouped by a bracket labeled "19".

Pno3: The third system shows Pno3's part. It features a treble clef, a bass clef, and a bass clef. The music includes eighth-note pairs and rests. Measure 166 starts with a treble clef, followed by a bass clef, and then a bass clef. Measure 167 begins with a bass clef. Measure 168 starts with a bass clef. Measures 166-168 are grouped by a bracket labeled "19". Measures 169-170 are grouped by a bracket labeled "19".

168

Pno1

Pno2

Pno3

Rings of Saturn

170

This musical score page shows two staves. The top staff is for 'Pno1' and the bottom staff is for 'Pno3'. Both staves begin at measure 170. The Pno1 staff has a treble clef, a key signature of one sharp, and a common time signature. It features a series of eighth-note patterns. The Pno3 staff has a bass clef, a key signature of one sharp, and a common time signature. It features sustained notes and rests. Measure 19 is indicated by a bracket under both staves. Measure 47 is indicated by a bracket under both staves.

Pno1

Pno3

19

47

Rings of Saturn

171

Pno1

Pno2

Pno3

19

19

19

172 49

 Pno1 {

 }

 Pno2 {

 }

 Pno3 {

 }

Rings of Saturn

50

174

Pno1

Pno2

Pno3

19

19

19

Rings of Saturn

176 51

 Pno1 {

 Pno2 {

 Pno3 {

Rings of Saturn

178

Pno1

Pno3

19

Musical score for three pianos (Pno1, Pno2, Pno3) at measure 179. The score consists of three systems of staves, each representing a piano part. The top system, Pno1, has a treble clef and bass clef. The middle system, Pno2, has a treble clef and bass clef. The bottom system, Pno3, has a treble clef and bass clef. The score is divided into measures by vertical bar lines. Measure 179 starts with a dynamic of **19**. The music features various note heads, stems, and rests. Measure 180 begins with a dynamic of **19**, followed by a dynamic of **19**. Measure 181 begins with a dynamic of **19**. The score includes several rehearsal marks and performance instructions like *mf*.

Pno1

Pno2

Pno3

Rings of Saturn

182

Pno1

Pno2

Pno3

Rings of Saturn

185

Pno1

19

19

55

Pno2

19

19

mf

Pno3

19

Rings of Saturn

187

Pno1

Pno2

Pno3

19

19

19

Rings of Saturn

189 (p).
 Pno1
 - 19 -
 ♯
 Pno2
 - 19 -
 ♯
 Pno3
 - 19 -
 ♯

The musical score consists of three staves, each representing a piano (Pno1, Pno2, and Pno3). The score is divided into two measures. Measure 189 begins with a dynamic marking '(p)' and a sharp sign (♯) above the first staff. The first staff (Pno1) has a measure number '19' with a bracket below it. The second staff (Pno2) has a measure number '19' with a bracket below it. The third staff (Pno3) has a measure number '19' with a bracket below it. Measure 19 continues with a dynamic marking 'p' and a sharp sign (♯) above the first staff. The first staff (Pno1) has a measure number '57' with a bracket below it. The second staff (Pno2) has a measure number '19' with a bracket below it. The third staff (Pno3) has a measure number '19' with a bracket below it.

Rings of Saturn

Musical score for Pno1 (Piano 1) at measure 58, page 191. The score consists of three staves: Treble, Alto, and Bass. The key signature changes from B-flat major to A major (two sharps). Measure 58 starts with a dynamic of 19. The Treble staff has a sixteenth-note pattern starting with a bass note. The Alto staff has eighth-note patterns. The Bass staff has eighth-note patterns. Measures 59 and 60 continue with similar patterns, ending with a dynamic of 19.

Musical score for Pno2 (Piano 2) at measure 58, page 191. The score consists of three staves: Treble, Alto, and Bass. The key signature changes from B-flat major to A major (two sharps). Measure 58 starts with a dynamic of 19. The Treble staff has eighth-note patterns. The Alto staff has eighth-note patterns. The Bass staff has eighth-note patterns. Measures 59 and 60 continue with similar patterns, ending with a dynamic of 19.

Musical score for Pno3 (Piano 3) at measure 58, page 191. The score consists of three staves: Treble, Alto, and Bass. The key signature changes from B-flat major to A major (two sharps). Measure 58 starts with a dynamic of 19. The Treble staff has eighth-note patterns. The Alto staff has eighth-note patterns. The Bass staff has eighth-note patterns. Measures 59 and 60 continue with similar patterns, ending with a dynamic of 19.

Rings of Saturn

194

Pno1

Pno2

Pno3

ff

f

mf

v

19

19

19

Rings of Saturn

Musical score for three pianos (Pno1, Pno2, Pno3) at measure 60, page 197.

The score consists of three systems of piano staves:

- Pno1:** The top system. It starts with a sixteenth-note pattern (19) followed by a rest. The right hand then plays a series of eighth-note pairs connected by slurs, ending with a grace note and a fermata over the next measure.
- Pno2:** The middle system. It begins with a sixteenth-note pattern (19), followed by a rest. The right hand then plays eighth-note pairs, with a sixteenth-note grace note in the first measure and a sustained bass note in the second measure.
- Pno3:** The bottom system. It starts with a sixteenth-note pattern (19), followed by a rest. The right hand then plays eighth-note pairs, with a sixteenth-note grace note in the first measure and a sustained bass note in the second measure.

Measure numbers 197, 19, and 19 are indicated above the staves to mark specific points in the music.

Rings of Saturn

201

Pno1

Pno2

Pno3

Rings of Saturn

205

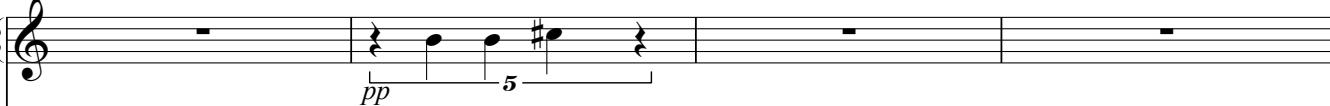
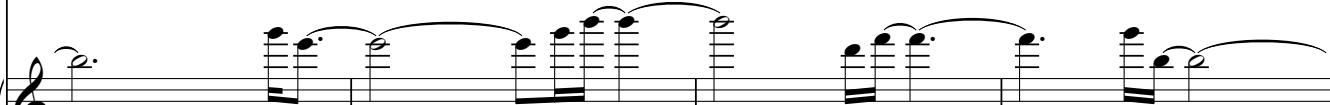
Pno1

Pno2

Pno3

Rings of Saturn

210

Pno2 {    

Pno3 {  



214

Pno2 {    

Pno3 {  

Rings of Saturn

Musical score for Pno1 and Pno2. The score consists of two systems of music. The top system, labeled 'Pno1', has two staves: treble and bass. The bottom system, labeled 'Pno2', also has two staves: treble and bass. Measure 1 (measures 218-219) is mostly rests. Measure 2 (measures 220-221) shows some activity, particularly in the bass staves. Measure 3 (measures 222-223) features a melodic line in the treble staff of Pno1. Measure 4 (measures 224-225) is mostly rests. Measure 5 (measures 226-227) shows a melodic line in the treble staff of Pno2.

Musical score for Pno2. This section contains only one system of music. It shows a single staff in treble clef. Measure 1 (measures 228-229) has a bracket under the first five notes. Measures 2 through 5 (measures 230-233) are mostly rests.

Musical score for Pno3. The score consists of two systems of music. The top system, labeled 'Pno3', has two staves: treble and bass. The bottom system, also labeled 'Pno3', has two staves: treble and bass. Measure 1 (measures 234-235) shows eighth-note patterns in the treble staves. Measure 2 (measures 236-237) shows sixteenth-note patterns in the treble staves. Measure 3 (measures 238-239) shows eighth-note patterns in the bass staves. Measure 4 (measures 240-241) shows eighth-note patterns in the treble staves. Measure 5 (measures 242-243) shows eighth-note patterns in the bass staves.

Rings of Saturn

222

Pno1

65

Pno3

Rings of Saturn

Musical score for piano parts Pno1 and Pno3 at measure 66, tempo 226. The score consists of two systems of four staves each.

Pno1 (Top System):

- Measures 1-3: Rests throughout.
- Measure 4: Bass staff has a bass note with a sharp sign. Treble staff has a bass note with a sharp sign.
- Measure 5: Bass staff has a bass note with a sharp sign. Treble staff has a bass note with a sharp sign.

Pno3 (Bottom System):

- Measures 1-3: Rests throughout.
- Measure 4: Bass staff has eighth-note pairs. Treble staff has eighth-note pairs.
- Measure 5: Bass staff has eighth-note pairs. Treble staff has eighth-note pairs.

230

Pno1

Pno2

5

Pno3

67

Rings of Saturn

234

Musical score for three pianos (Pno1, Pno2, Pno3) at measure 234.

Pno1 (Top Staves):

- Measures 1-4: Rests throughout.
- Measure 5: Starts with a sixteenth-note pattern: $\text{B}_\flat \text{A}^\sharp \text{B}_\flat \text{A}^\sharp$, followed by a eighth-note G , then a sixteenth-note pattern: $\text{B}_\flat \text{A}^\sharp \text{B}_\flat \text{A}^\sharp$.

Pno2 (Second Staff):

- Measures 1-4: Rests throughout.
- Measure 5: Starts with a sixteenth-note pattern: $\# \text{E} \text{D} \text{C}$, followed by a eighth-note C , then a sixteenth-note pattern: $\# \text{E} \text{D} \text{C}$.

Pno3 (Bottom Staves):

- Measures 1-4: Starts with a sixteenth-note pattern: $\text{F} \text{E} \text{D}$, followed by a eighth-note D , then a sixteenth-note pattern: $\text{F} \text{E} \text{D}$.
- Measure 5: Starts with a sixteenth-note pattern: $\text{B}_\flat \text{A}^\sharp \text{B}_\flat \text{A}^\sharp$, followed by a eighth-note A^\sharp , then a sixteenth-note pattern: $\text{B}_\flat \text{A}^\sharp \text{B}_\flat \text{A}^\sharp$.

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238

69

This musical score page contains two systems of music. The top system, labeled 'Pno1', consists of four staves: Treble, Alto, Bass, and another Bass staff. The bottom system, labeled 'Pno3', also consists of four staves: Treble, Alto, Bass, and another Bass staff. Measure 238 begins with a rest in all voices. The first half of the measure features various melodic lines and harmonic patterns, including a prominent treble clef staff with sixteenth-note patterns and bass clef staves with eighth-note patterns. Measure 239 continues with similar melodic and harmonic complexity, maintaining the four-staff format for each system.

70

242

Pno1

Pno3

Rings of Saturn

246

Pno1

Pno2

Pno3

Rings of Saturn

251

Pno1

Pno3

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