Orbital Resonance

for three retuned, computer-driven pianos

by Kyle Gann

2015

Orbital Resonance (2015)

When the New Horizons spacecraft took its historic photos of Pluto in July 2015, there was a lot written about Pluto. I learned, for instance, for the first time, that although the orbits of Pluto and Neptune overlap, they are prevented from colliding by the 2-to-3 ratio in their rotations around the sun; Pluto goes around the sun in 247.94 earth years, and Neptune in 164.8, and 247.94/164.8 equals 1.50449.... This kind of mutually influenced periodicity, as it turns out (how was I an astrologer for thirty years without learning this?), is common among pairs, trios, quadruples of planets, moons, asteroids, and so on, and is called *orbital resonance*. Three of the moons of Jupiter exhibit rotational ratios of 1:2:4, and there's even an asteroid that has a 5:8 dance going with respect to the earth. This is truly the harmony of the spheres, the surprisingly simple mathematical relations that planets in a rotational system fall into in response to each other's gravity.

Chalk it up to my personal eccentricities that this suddenly gave me a new way to compose. For 35 years I'd been writing repeating cycles at different tempos, and it has sometimes been an aesthetic problem for me when the articulation points of those cycles coincide by chance. But the solution, as it turned out, was already in our stars. Inspired by this new knowledge, I started using much simpler ratios than I had been using (3:4, 5:6:7 instead of 17:19:23), but shifting each one a slight amount so that the articulated beats would *never* coincide. It gave me a new way to create melody from the articulated beats among the different cycles. I immediately started a piece titled *Orbital Resonance*.

Orbital Resonance is for three computer-controlled pianos (Disklaviers, for instance), retuned to include 33 pitches to the octave, the result of eight harmonic series' on the first eight odd-numbered harmonics of Eb. Although the piece is fairly continuous within its moment form, its successive panels fall into six sections whose progression makes the derivation of the characteristic rhythm increasingly clear:

> 1. Articulation of the characteristic rhythm by various pitches in the scale less than a quarter-tone apart, with harmonizations (taking advantage of where such pitches in the scale differ by less than a quarter-tone) (mm. 1-103).
>
> 2. Articulation of the characteristic rhythm by dyads from different harmonic series' (mm. 103-132).
>
> 3. The characteristic rhythm fused into a single melody, accompanied by chords outlining the rhythmic derivation (mm. 133-191).
>
> 4. Articulation of the characteristic rhythm by widely-spaced sonorities separated by extremely parsimonious voice-leading (expansion of the 1st section and 2nd section ideas) (mm. 192-249).
>
> 5. Articulation of the characteristic rhythm divided out among increasingly audible independent melodic ostinatos (expansion of the 3rd section idea) (mm. 249-273).

6. A coda returning to the initial idea, with sparser harmonization (mm. 274-301).

This is a kind of broken symmetry characteristic of my music: the first section is paralleled with the sixth, the fourth combines the first and second, and the fifth expands on the third. I provide the plan not to suggest that the piece should be heard in a corresponding way, but merely to draw attention to the presence of an internal logic that might not be immediately evident.

For years I'd been trying to write something more elaborate both microtonally and polyrhythmically (and polytonally) than *Custer and Sitting Bull* (1999), and this is it: Nancarrow fused with Ben Johnston and La Monte Young with a dash of *Piano Phase* thrown in. And as with *Custer*, I've dedicated it to Ben, who in 1984 started me down this incredibly labor-intensive road.

- Kyle Gann
August, 2015
Germantown, NY

Technical Specifications

The 33-pitch tuning of the three pianos (the same in every octave) is as follows, given first in the number of cents above E-flat, and then as ratios to the E-flat 1/1:

Piano	1		2		3	
D	1088	15/8	977	225/128	1044	117/64
Db	969	7/4	938	55/32	906	27/16
C	857	105/64	773	25/16	840	13/8
B	738	49/32	755	99/64	729	195/128
Bb	702	3/2	590	45/32	609	91/64
A	551	11/8	551	11/8	481	169/128
Ab	471	21/16	440	165/128	408	81/64
G	386	5/4	320	77/64	342	39/32
Gb	204	9/8	275	75/64	275	75/64
F	155	35/32	192	143/128	192	143/128
E	92	135/128	53	33/32	27	65/64
Eb	0	1/1	1103	121/64	1173	63/32

Note that no string needs to be raised higher than its natural tuning except for the B-flat on piano 1, which is 2¢ sharp (or if one prefers, 2¢ could be subtracted from all quantities).

For electronic realization of the piece, it can prove helpful to reconfigure he tuning as a reference pitch in cycles per second for each piano, and ratios derived from that standard:

Tuning pitch:	38.891 cps	36.7641 cps	38.2833 cps
D	15/8	225/121	13/7
Db	7/4	20/11	12/7
C	105/64	200/121	104/63
B	49/32	18/11	65/42
Bb	3/2	180/121	13/9
A	11/8	16/11	169/126
Ab	21/16	15/11	9/7
G	5/4	14/11	26/21
F#	9/8	150/121	25/21
F	35/32	13/11	143/126
E	135/128	12/11	65/63
Eb	1/1	1/1	1/1

In the configuration of certain tuning softwares, the following sequences might facilitate getting the required tuning:

Piano 1:
38.891 = Eb0
1/1, 135/128, 35/32, 9/8, 5/4, 21/16, 11/8, 3/2, 49/32, 105/64, 7/4, 15/8

Piano 2:
36.7641485 = Eb0
1/1, 12/11, 13/11, 150/121, 14/11, 15/11, 16/11, 180/121, 18/11, 200/121, 20/11, 225/121

Piano 3:
38.283333 = Eb0
1/1, 65/63, 143/126, 25/21, 26/21, 9/7, 169/126, 13/9, 65/42, 104/63, 12/7, 13/7

For purposes of analysis, the entire scale (which I refer to as my 8x8 scale) is given below, grouping its pitches into eight harmonic series' on the 1^{st}, 3^{rd}, 5^{th}, 7^{th}, 9^{th}, 11^{th}, 13^{th}, and 15^{th} harmonics of E-flat, and naming each pitch in a typographical equivalent of Ben Johnston's just-intonation notation:

Pitch name	Ratio	Cents	1/1	3/2	5/4	7/4	9/8	11/8	13/8	15/8
Db^^-	121/64	1103						11		
D	15/8	1088	15	5	3					1
Db13	117/64	1044					13		9	
C#+	225/128	977								15
Db7	7/4	969	7			1				
C^	55/32	938			11			5		
C+	27/16	906		9			3			
C7+	105/64	857				15				7
Cb13	13/8	840	13						1	
B	25/16	773			5					
Bb^	99/64	755					11	9		
Cb77+	49/32	738				7				
Bb13	195/128	729							15	13
Bb	3/2	702	3	1						
Bbb713	91/64	609				13			7	
A+	45/32	590		15	9		5			3
Ab^	11/8	551	11					1		
Abb1313	169/128	481							13	
Ab7+	21/16	471		7		3				
G^	165/128	440						15		11
G+	81/64	408					9			
G	5/4	386	5		1					
Gb13	39/32	342		13					3	
Gb7^	77/64	320				11		7		
F#+	75/64	275			15					5
F+	9/8	204	9	3			1			
Fb13^	143/128	192						13	11	
F7+	35/32	155			7	5				
E+	135/128	92					15			9
Eb^	33/32	53		11				3		
Eb13	65/64	27			13				5	
Eb	1/1	0	1							
Eb7+	63/32	1173				9	7			

In Johnston's notation, + raises a pitch by 81/80, # raises it by 25/24, b lowers it by 24/25, 7 lowers it by 35/36, ^ raises it by 33/32, 13 raises it by 65/64, and F-A-C, C-E-G, and G-B-D are all perfectly tuned 4:5:6 major triads.

to Ben Johnston

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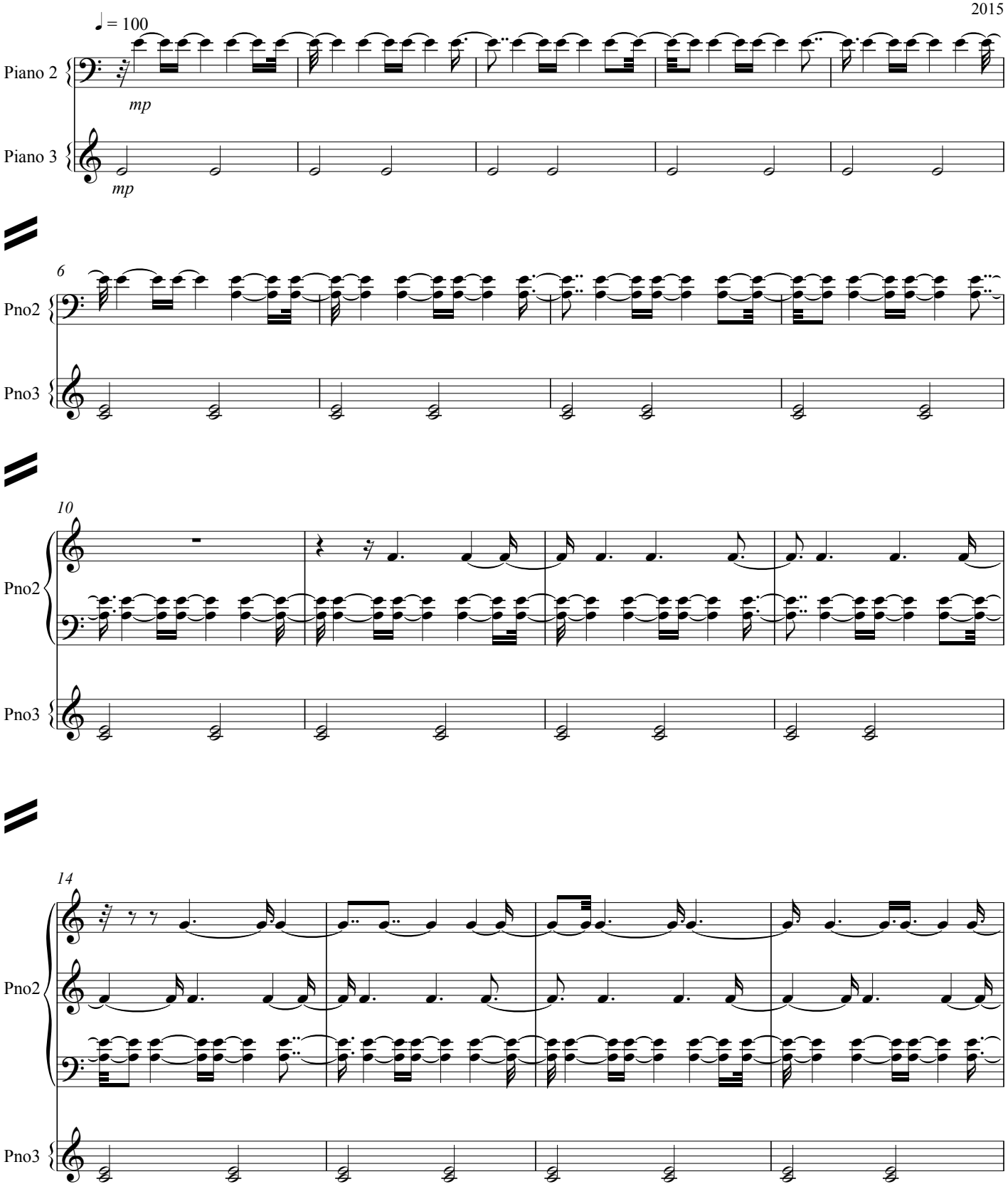
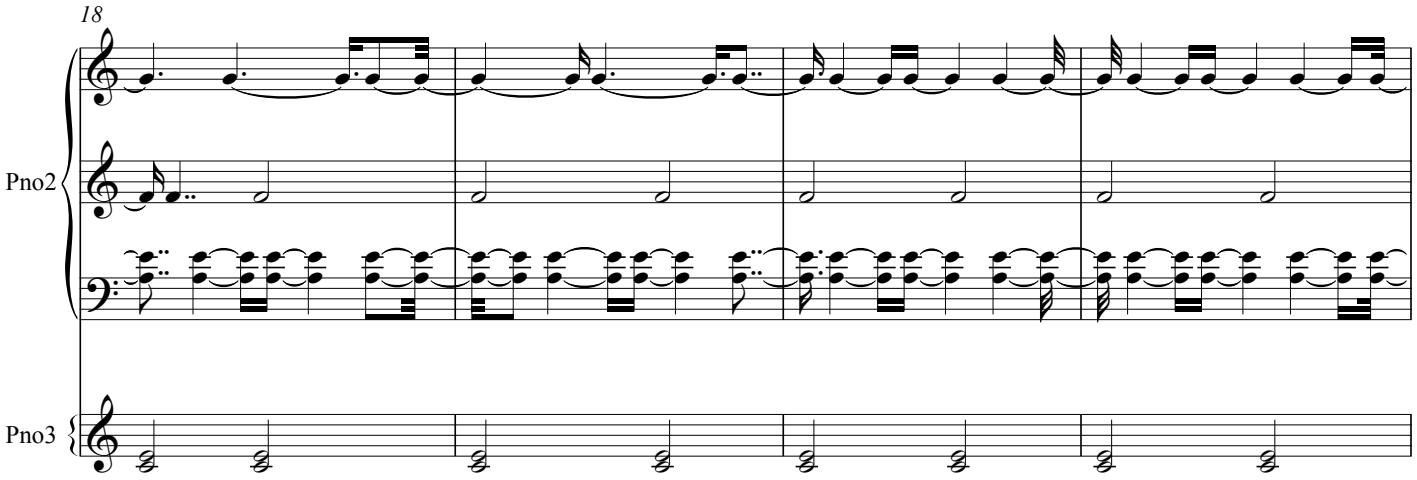
Kyle Gann
2015

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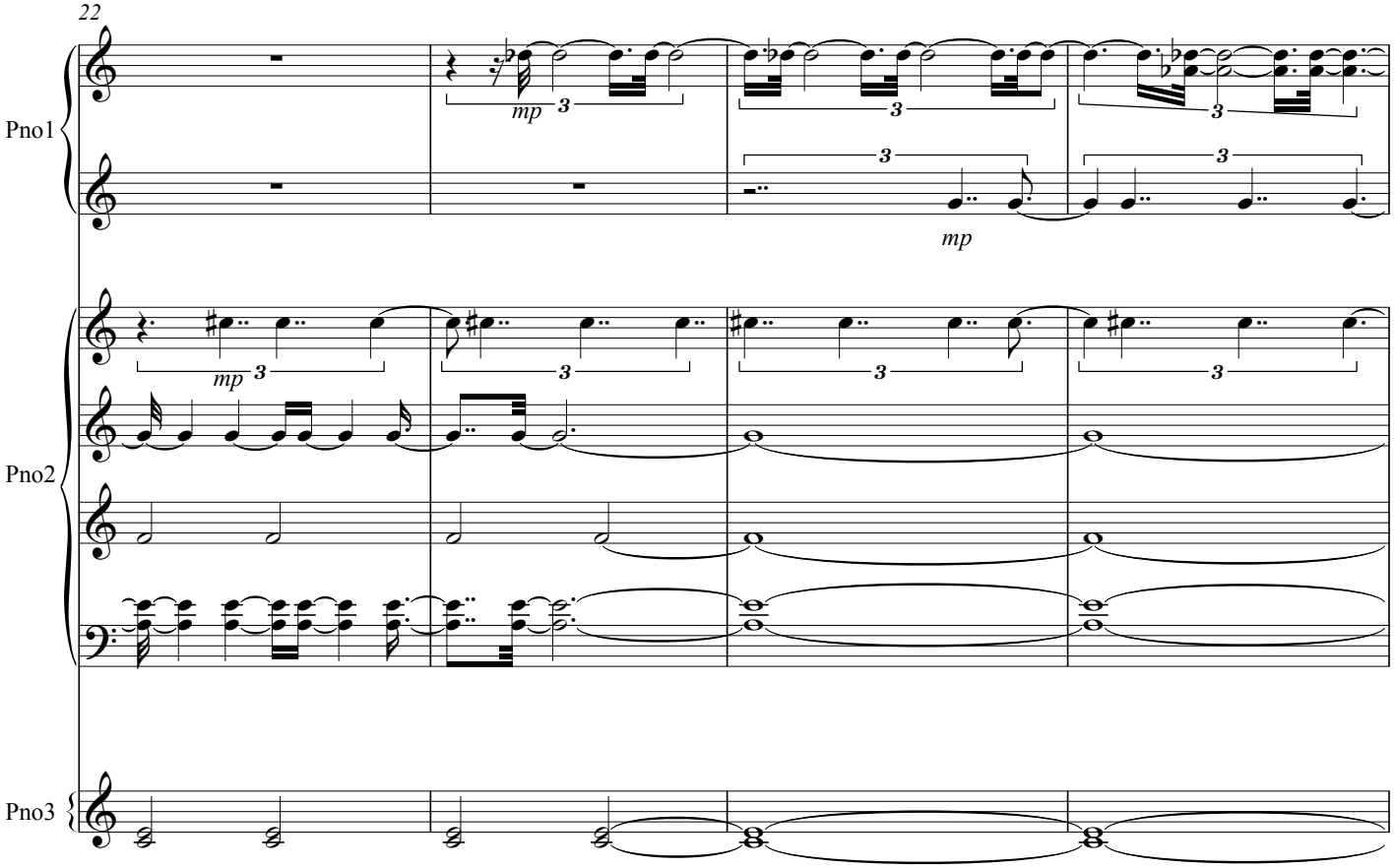
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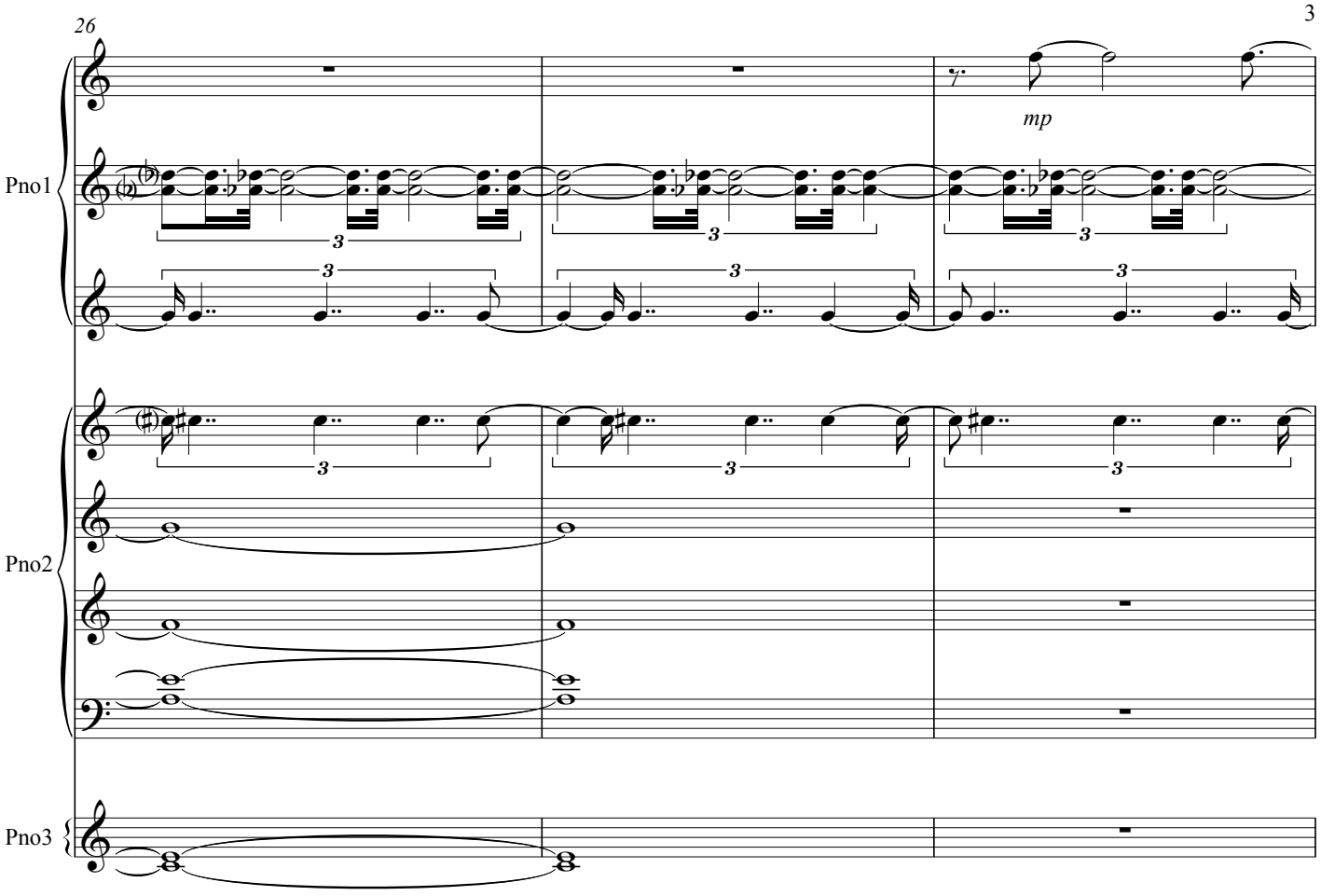
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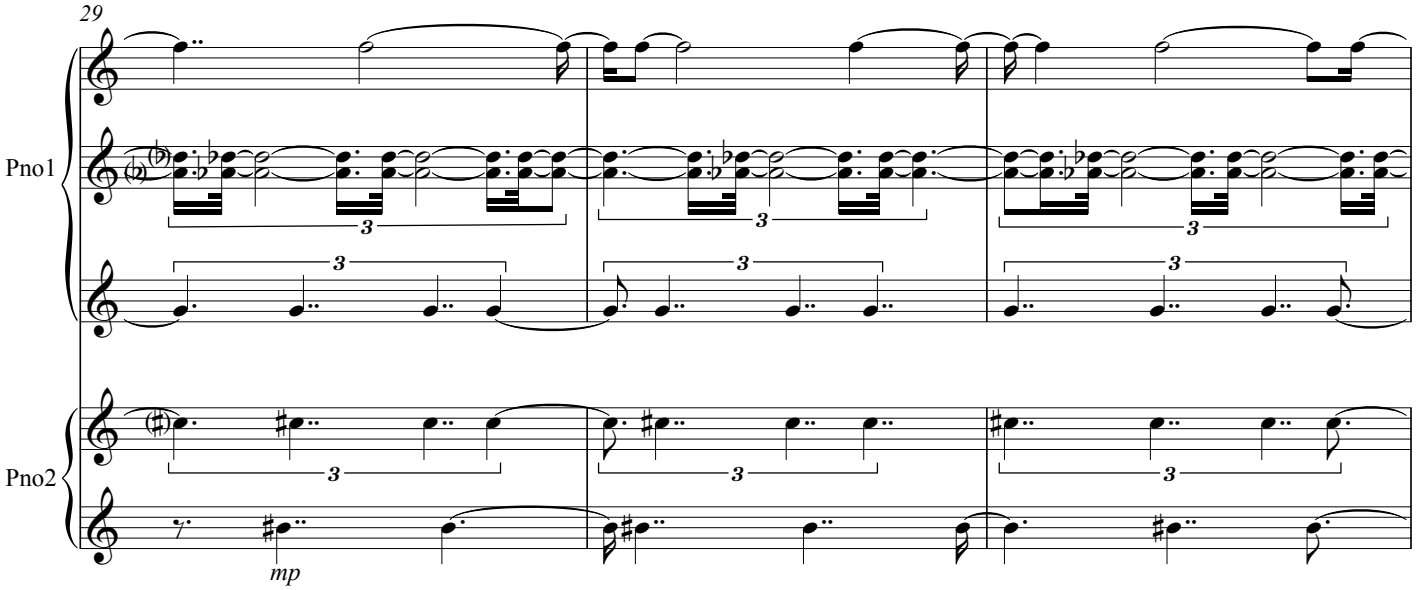
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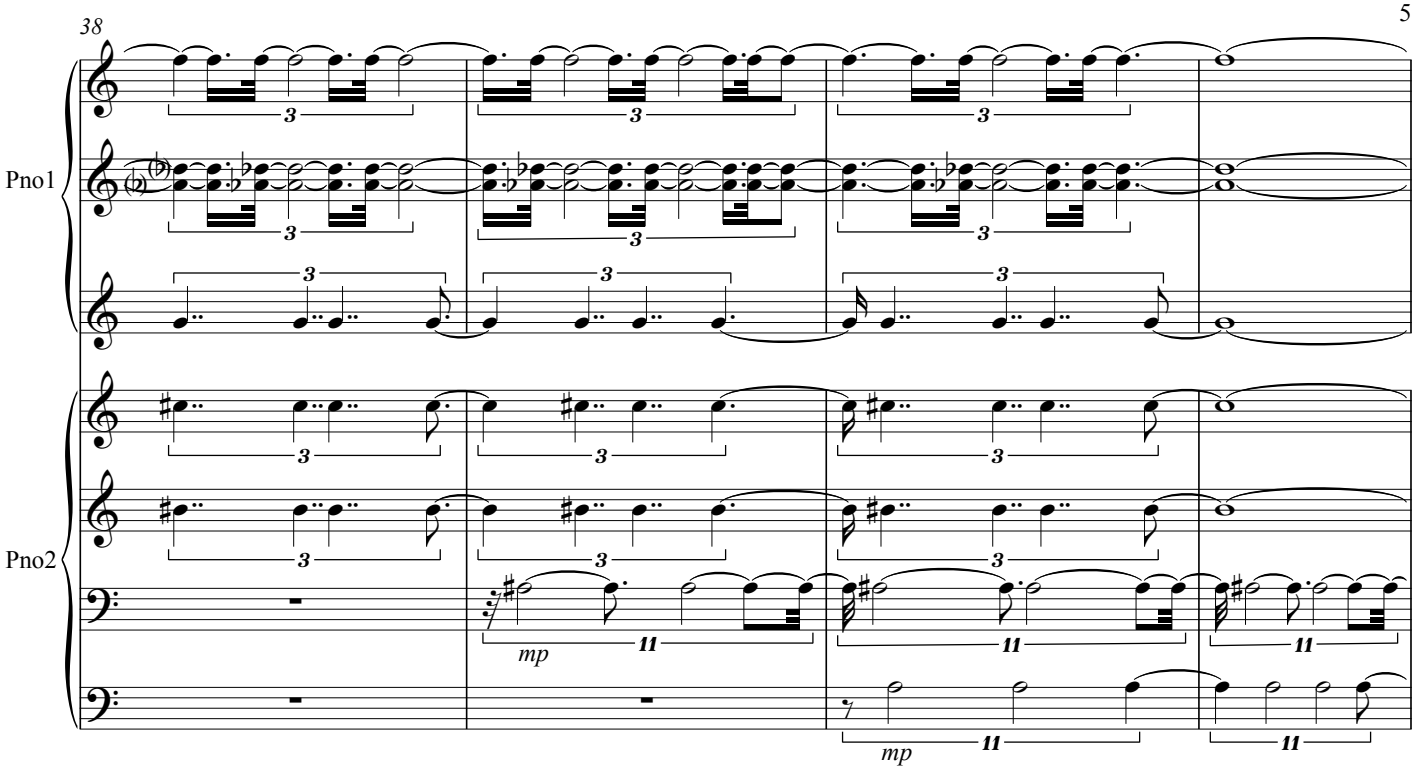
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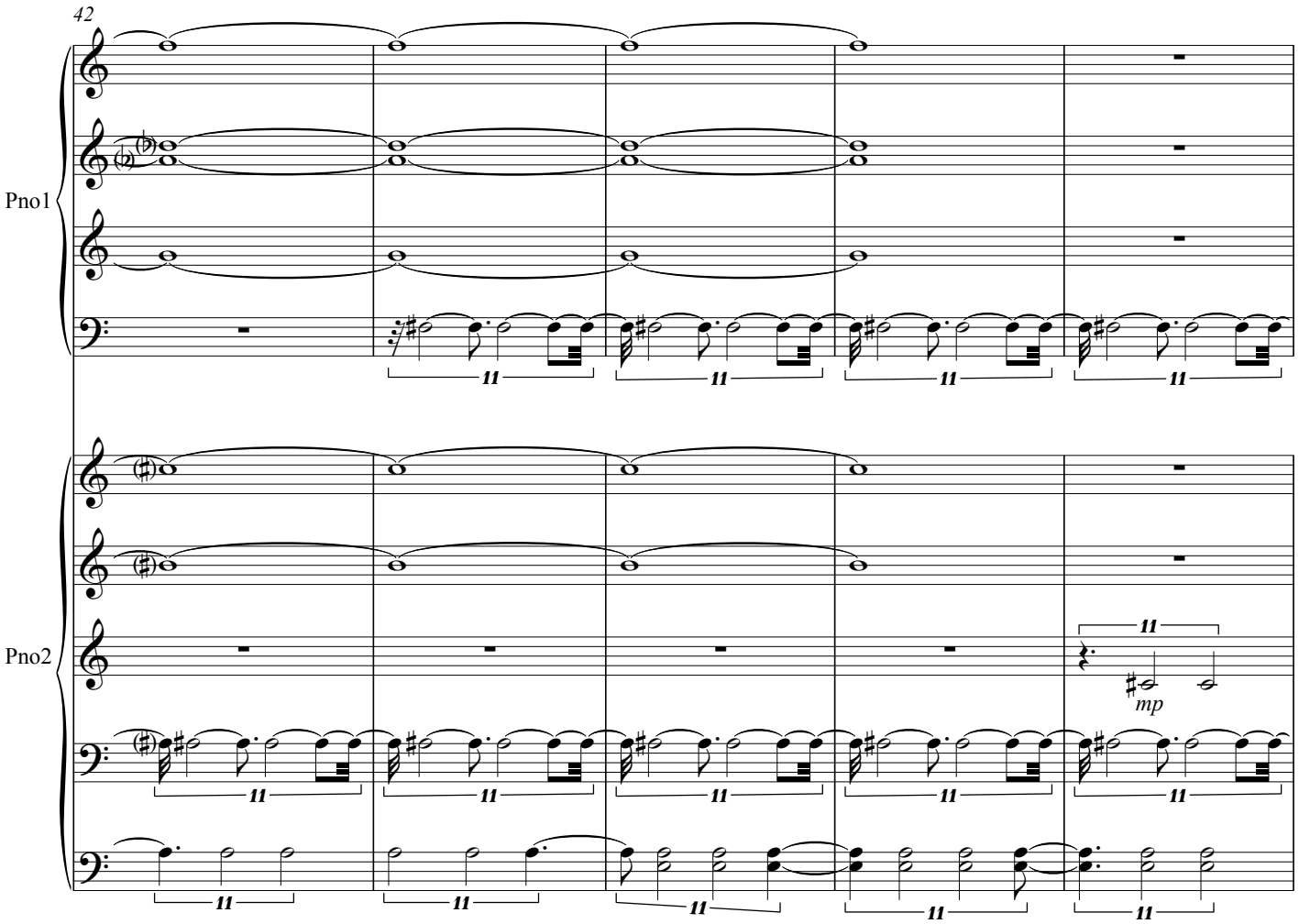
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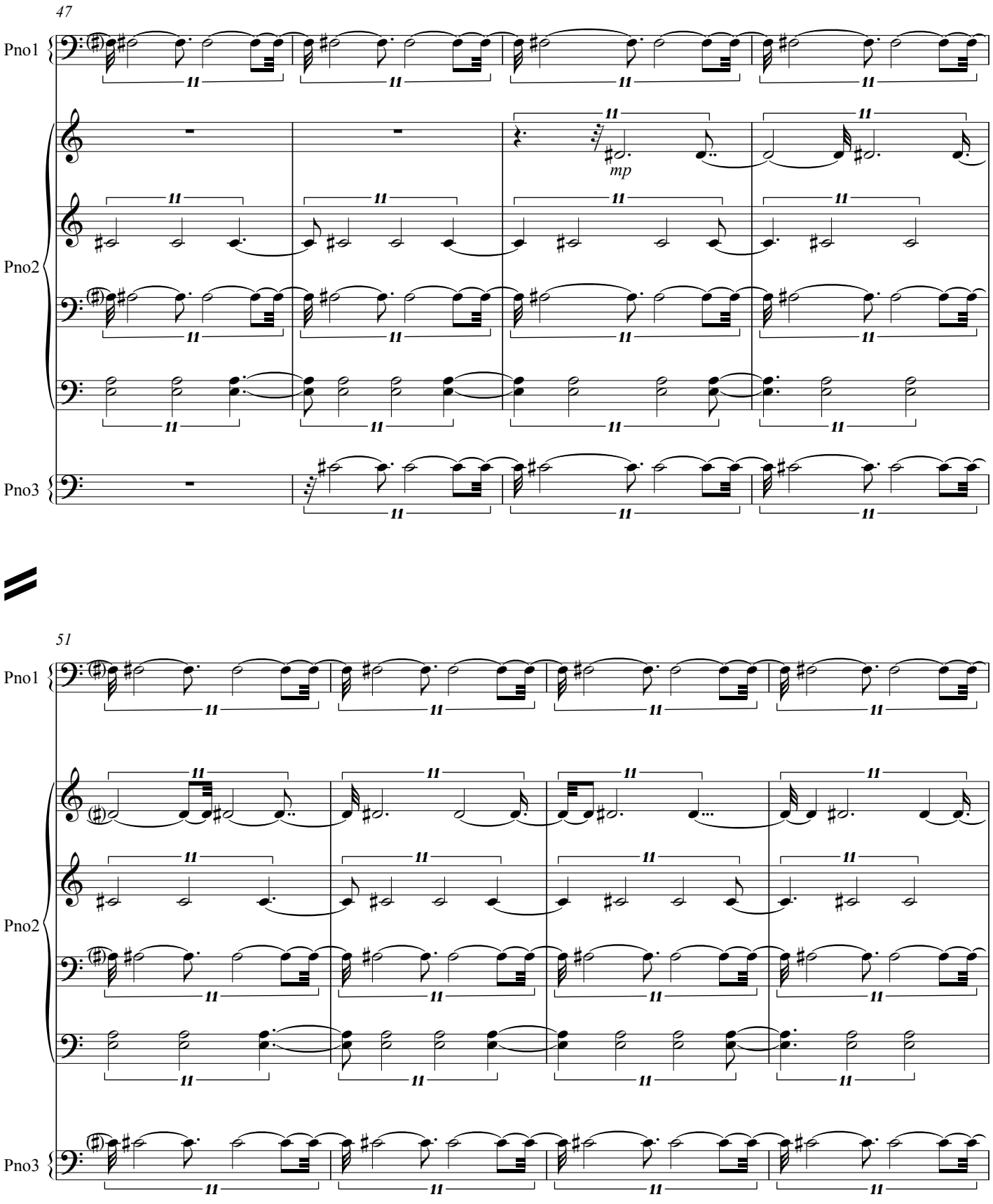
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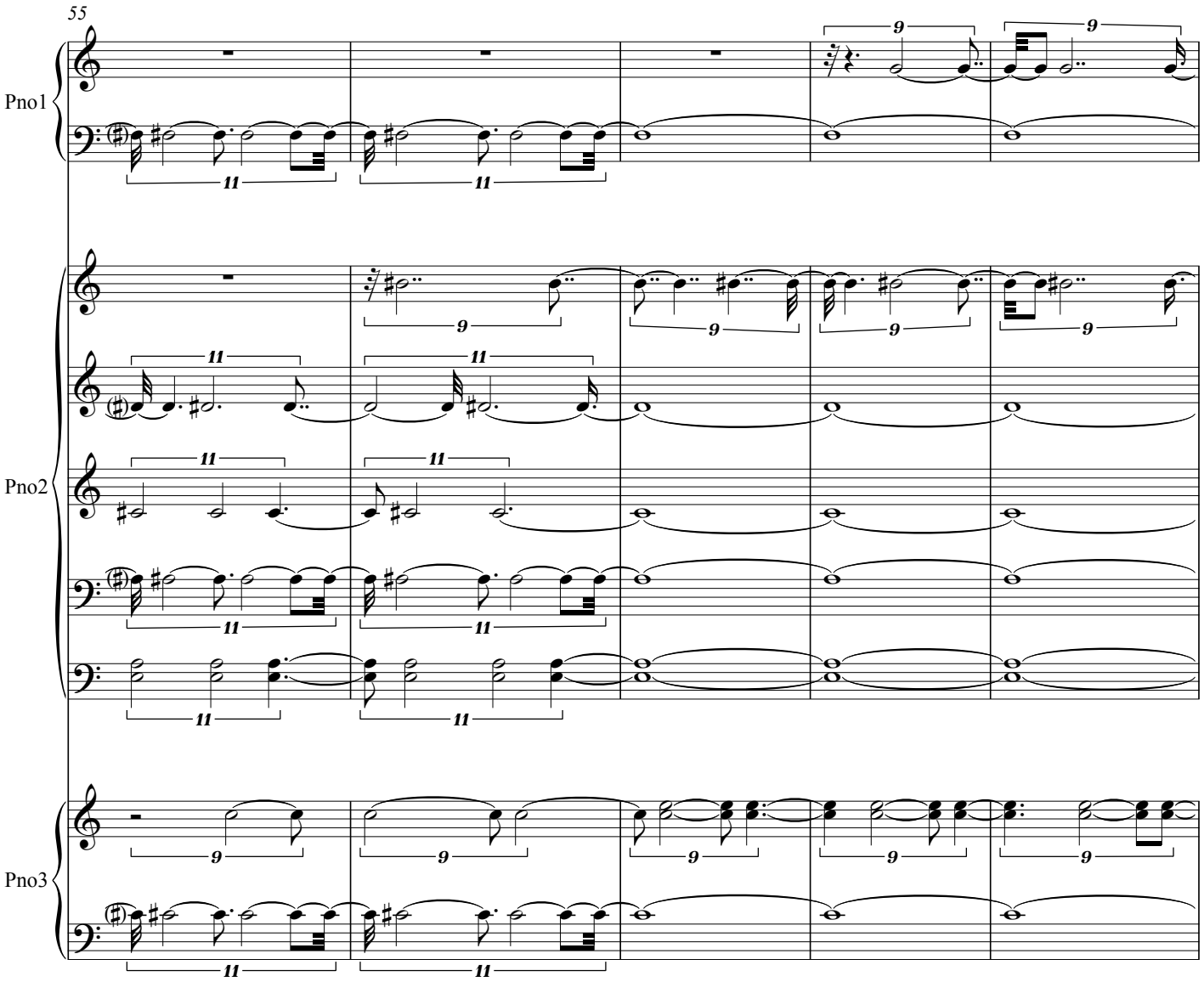
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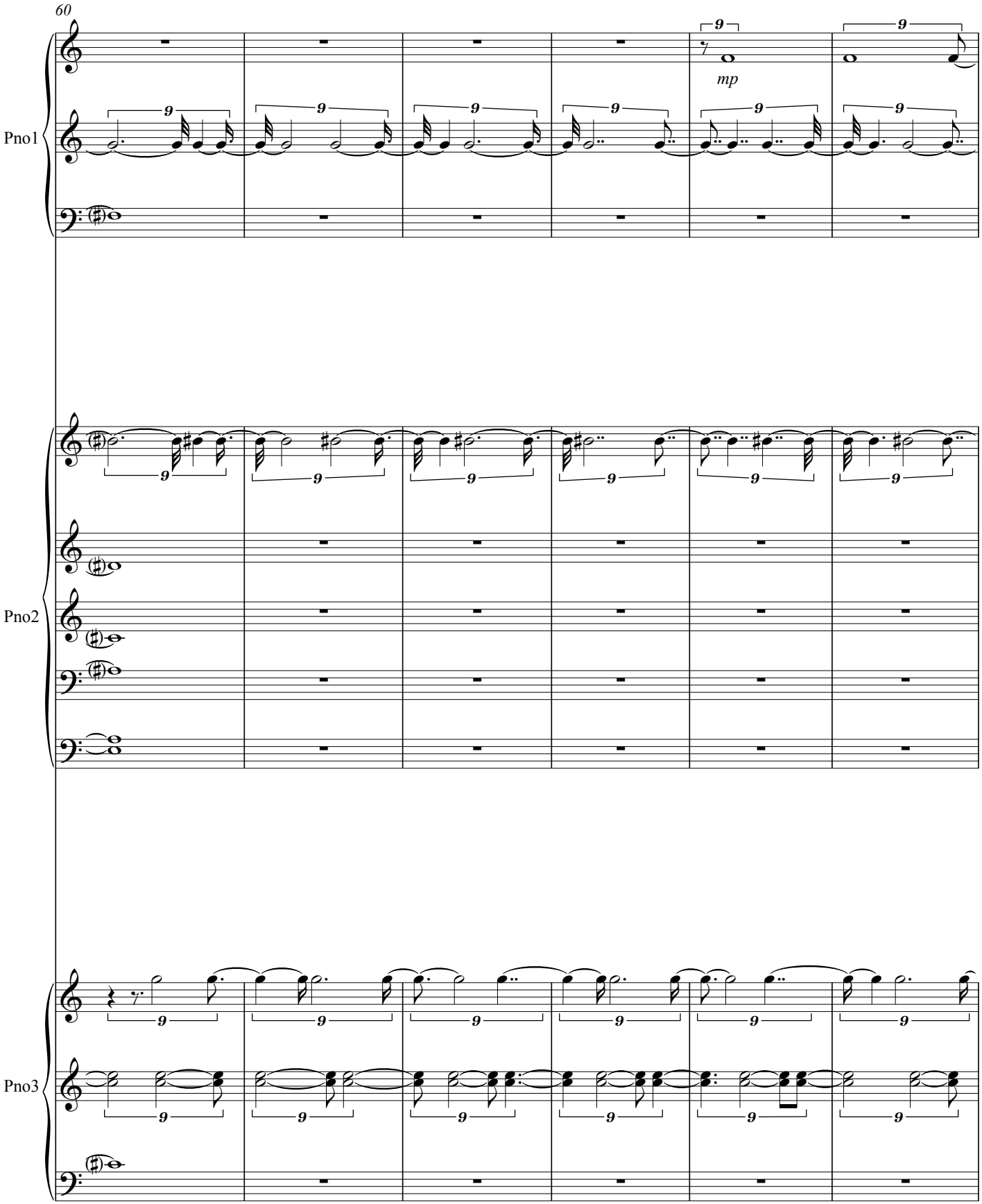
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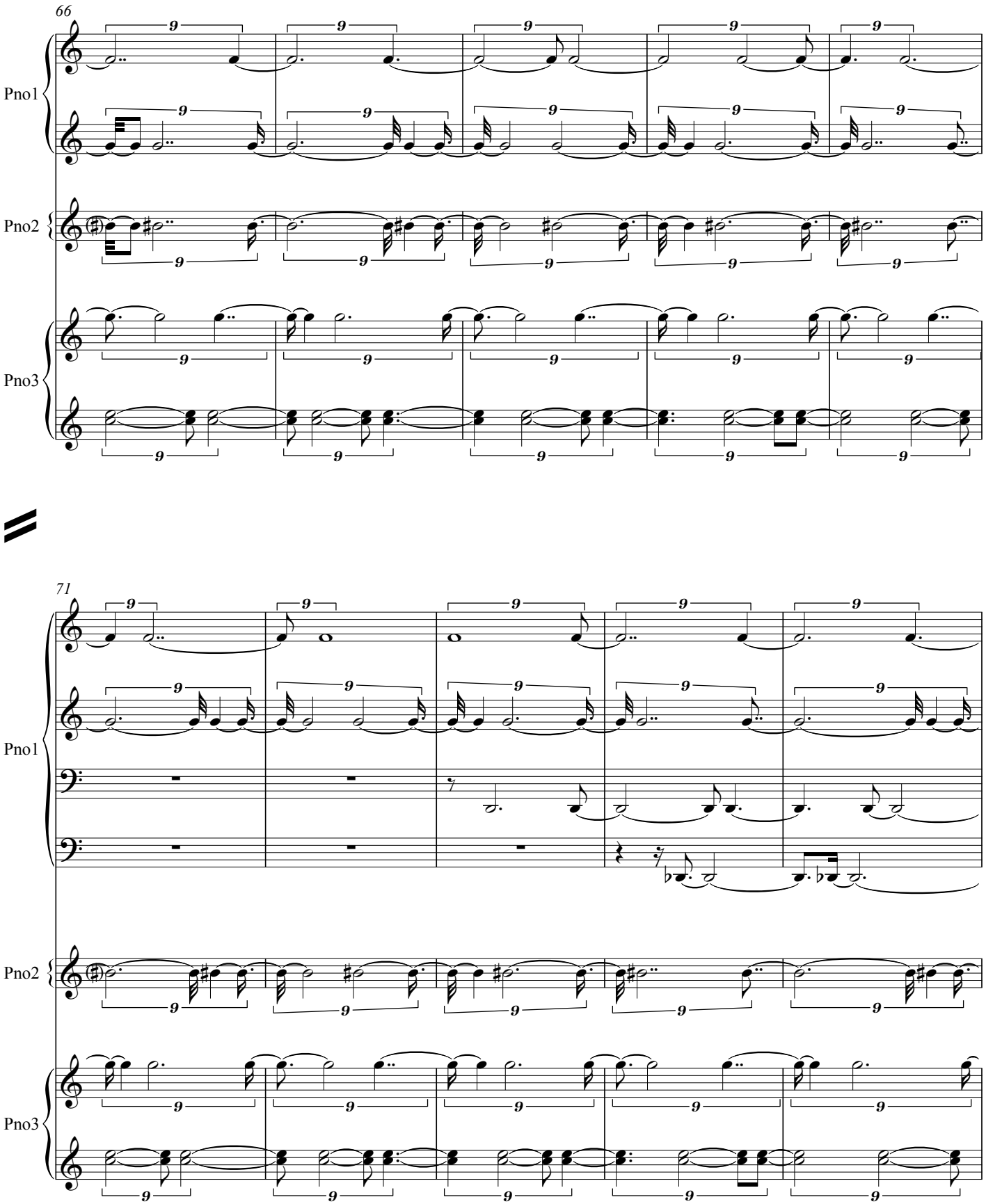
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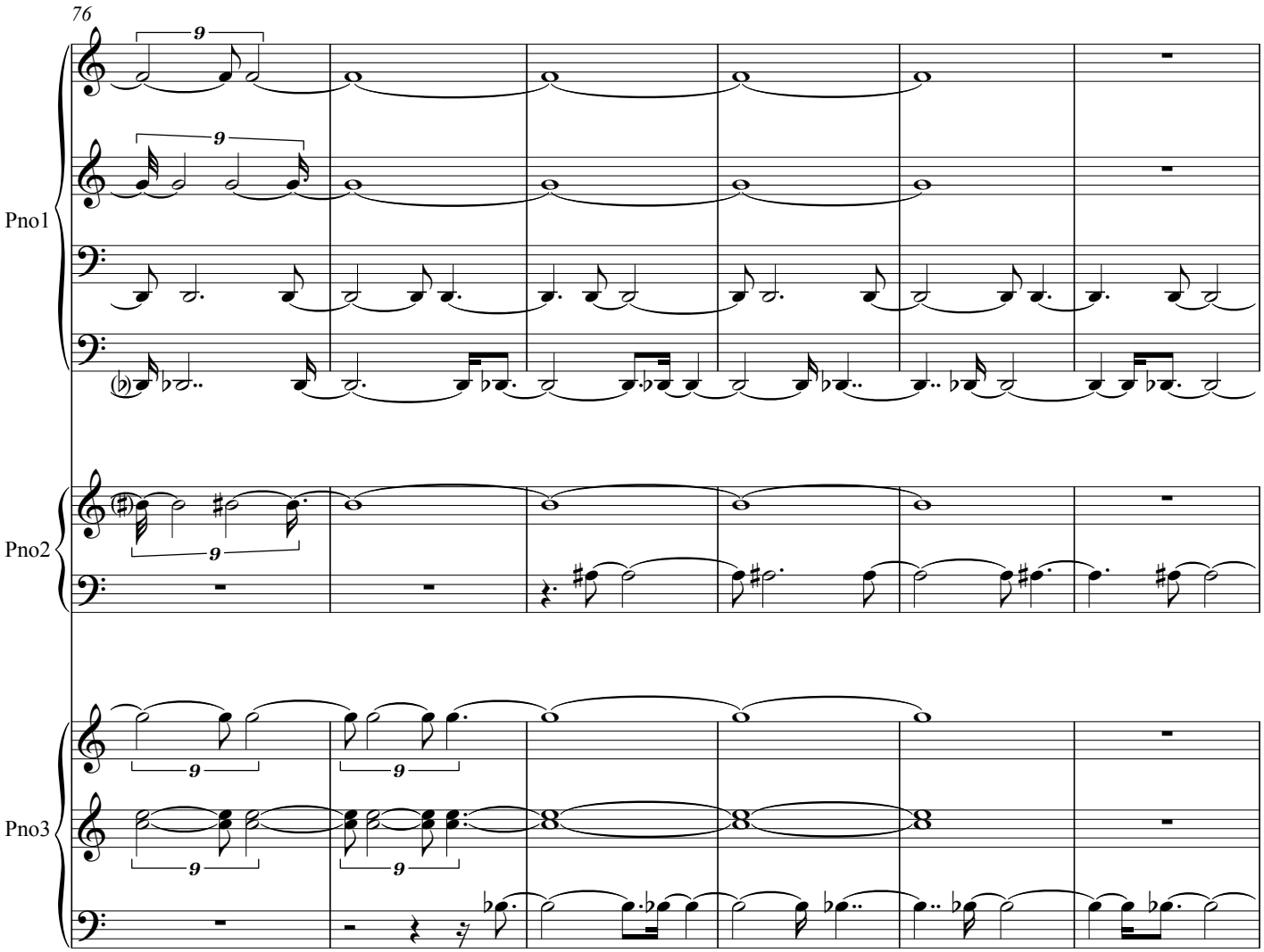
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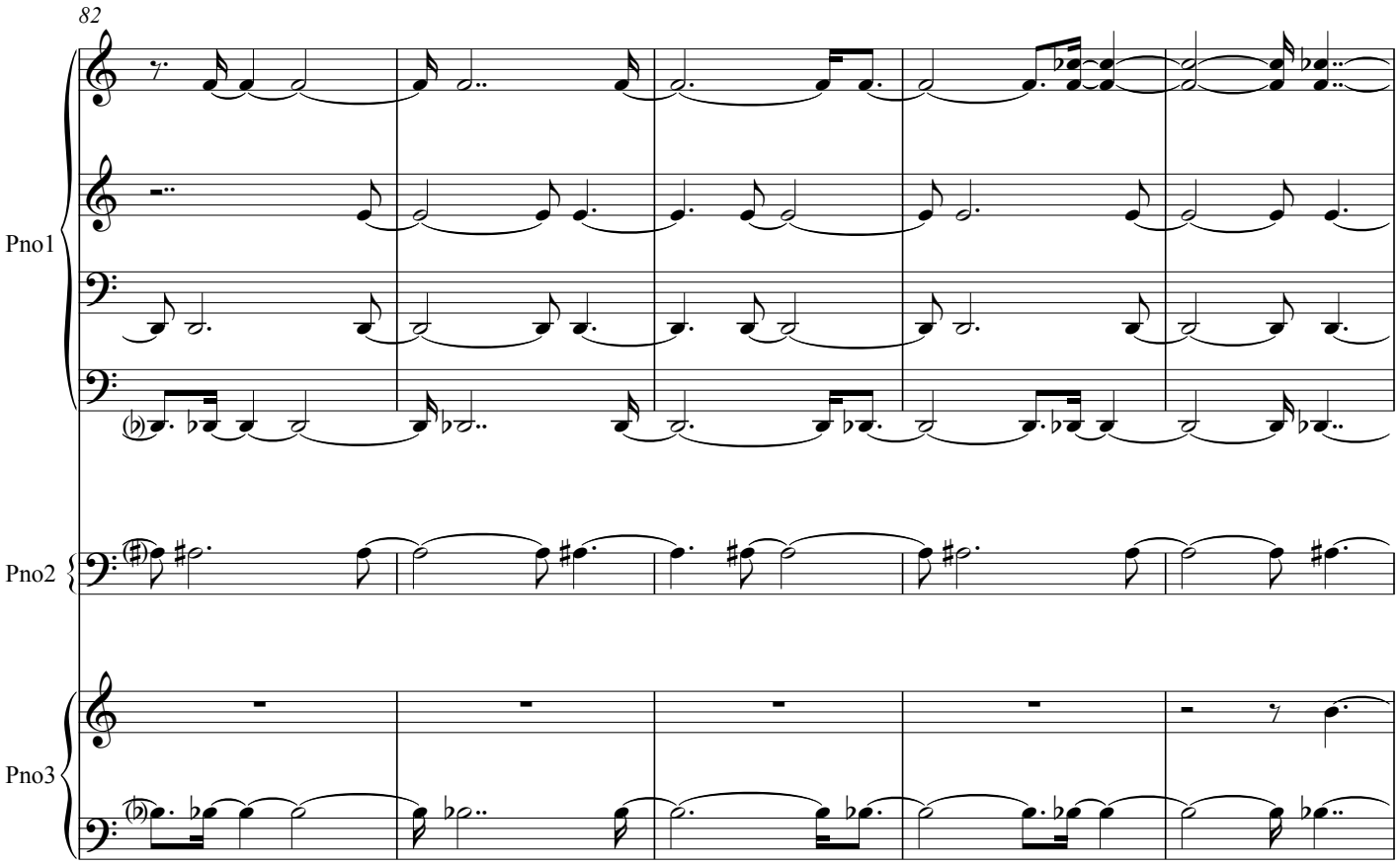
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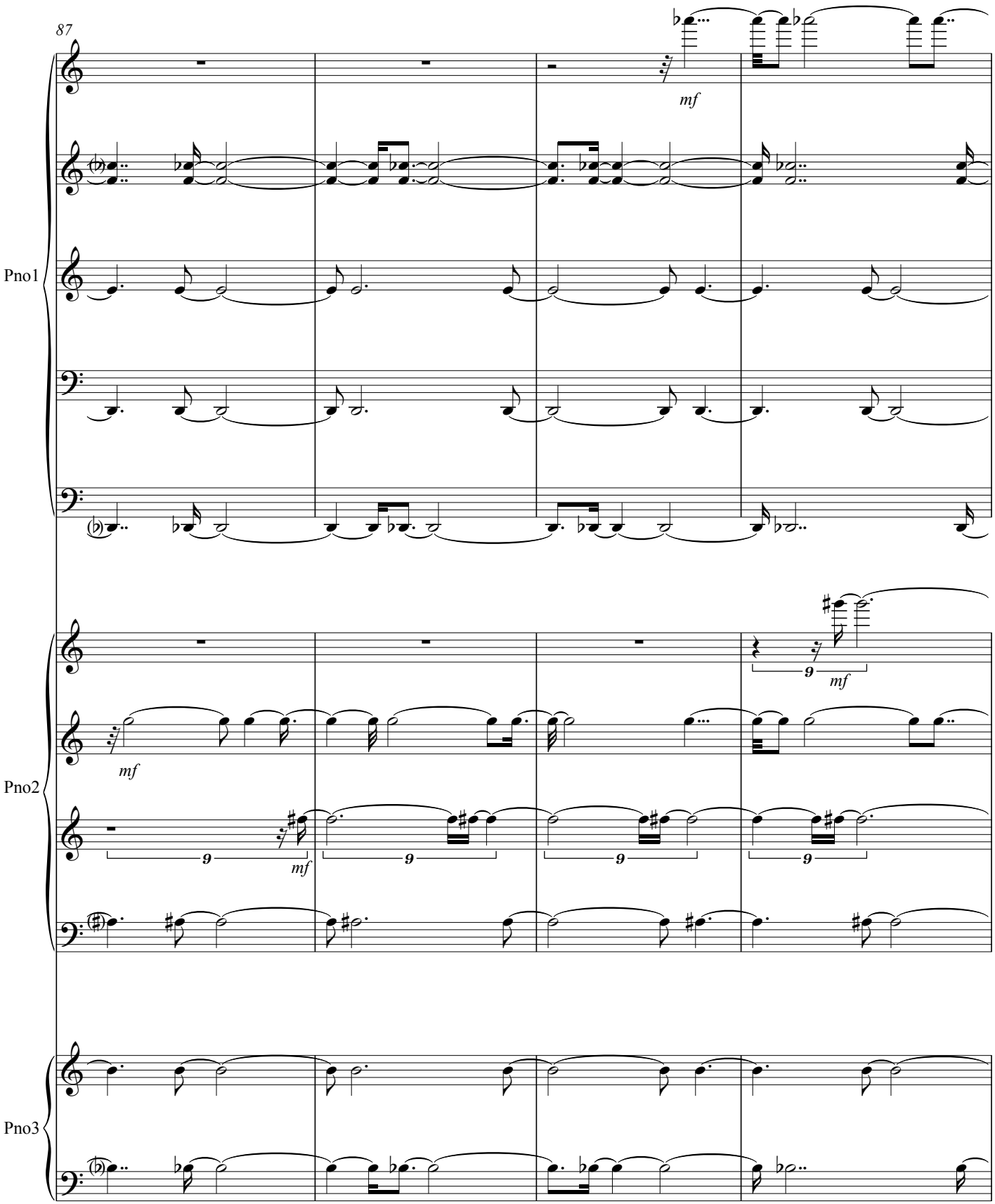
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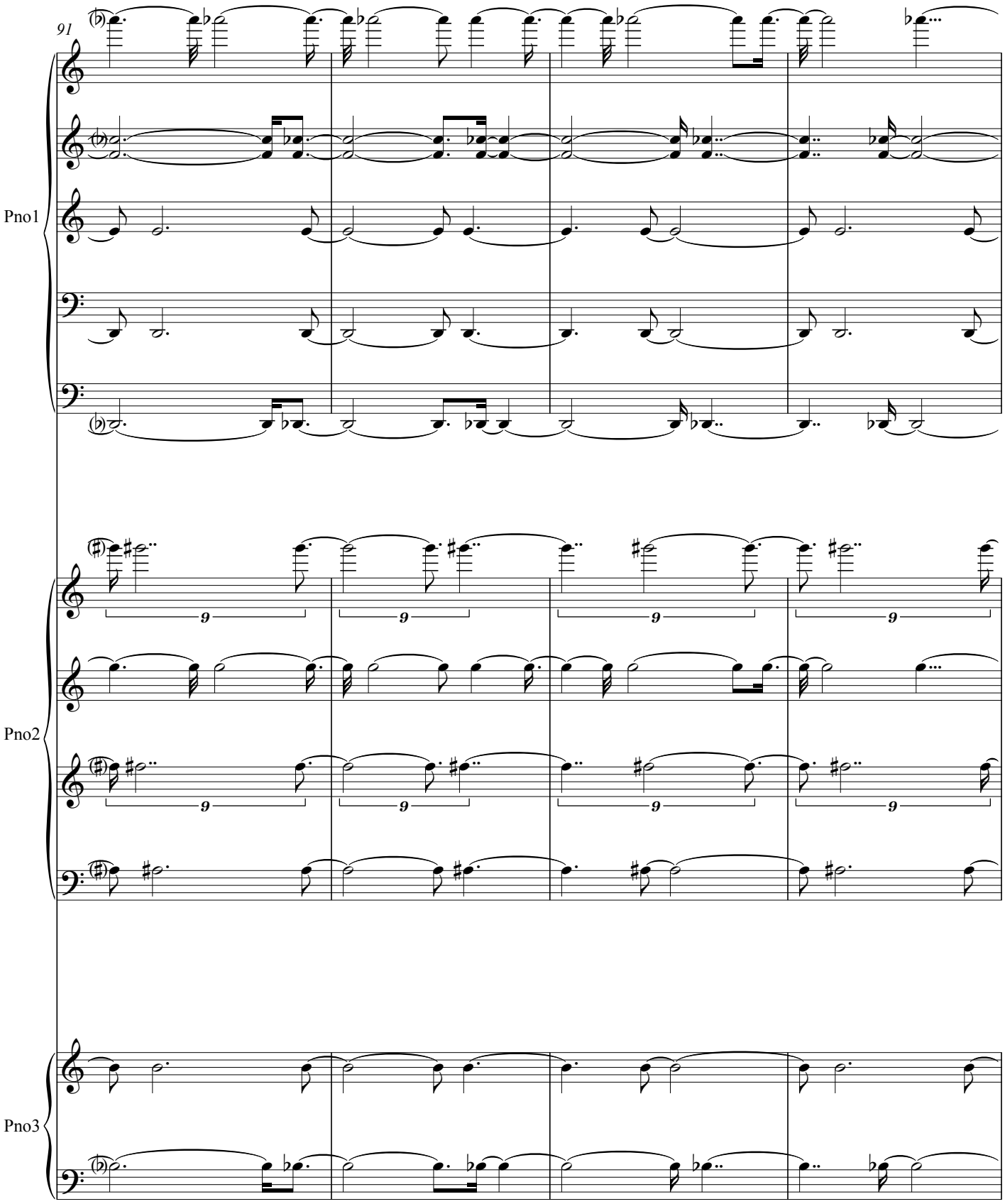
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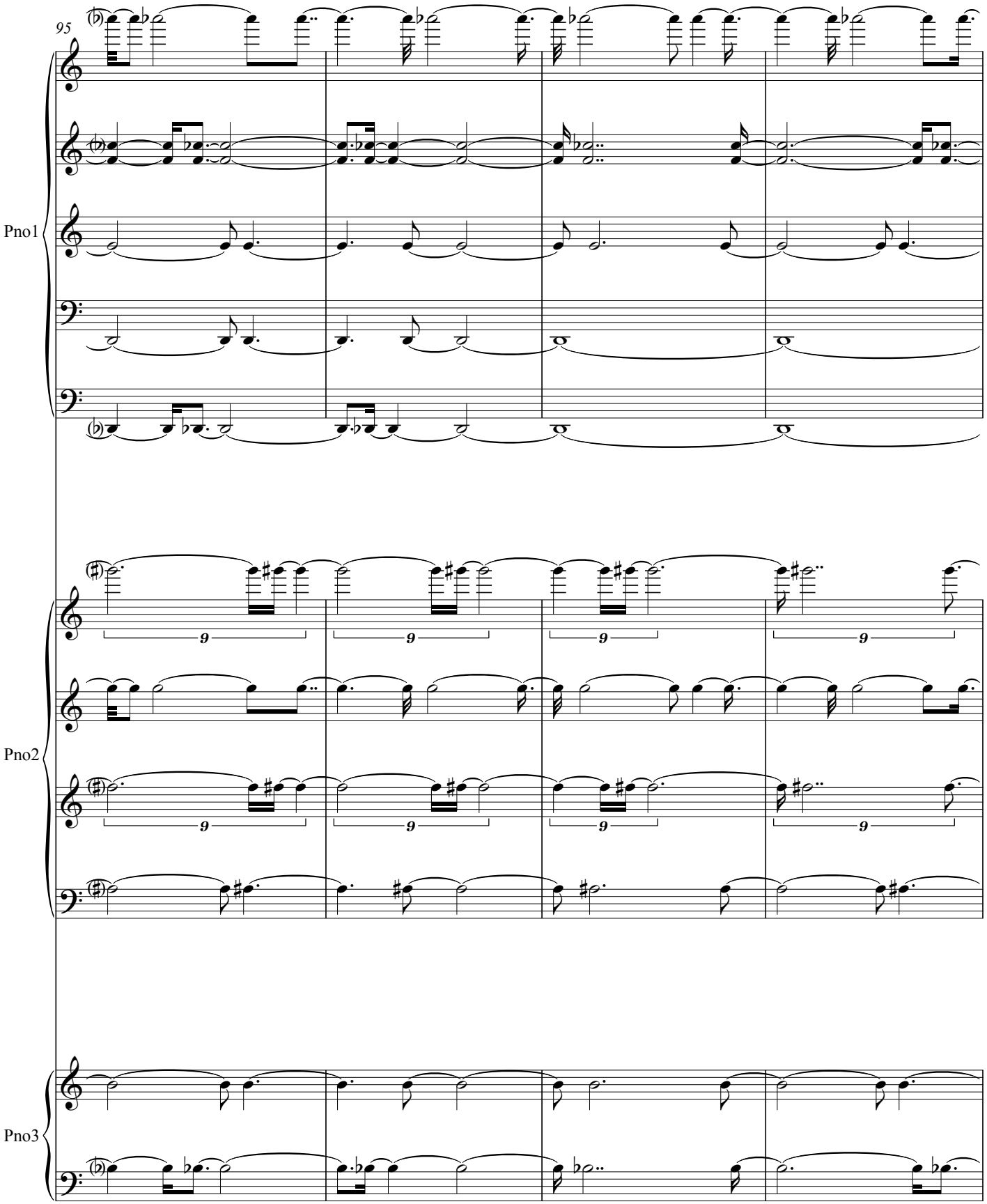
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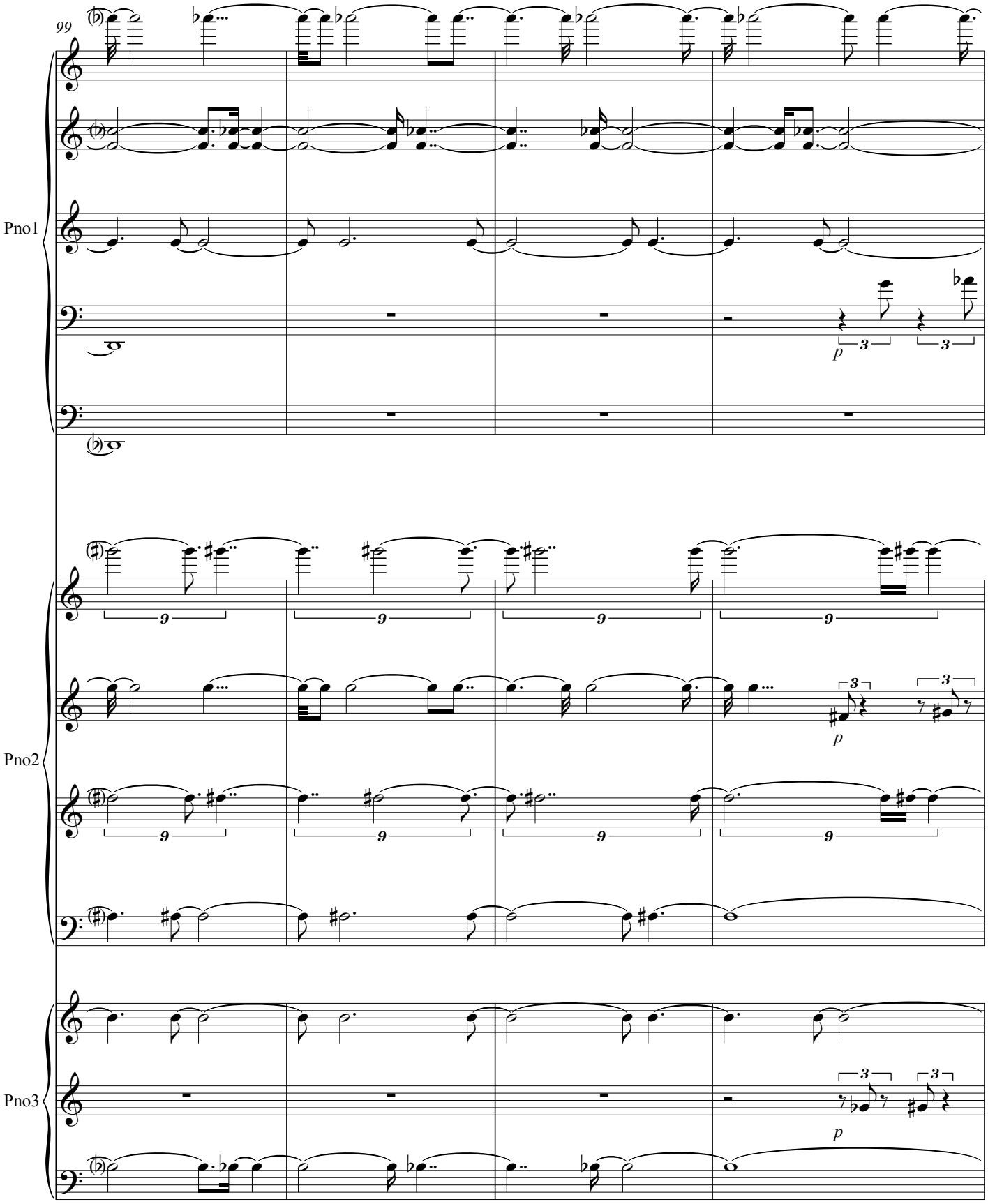
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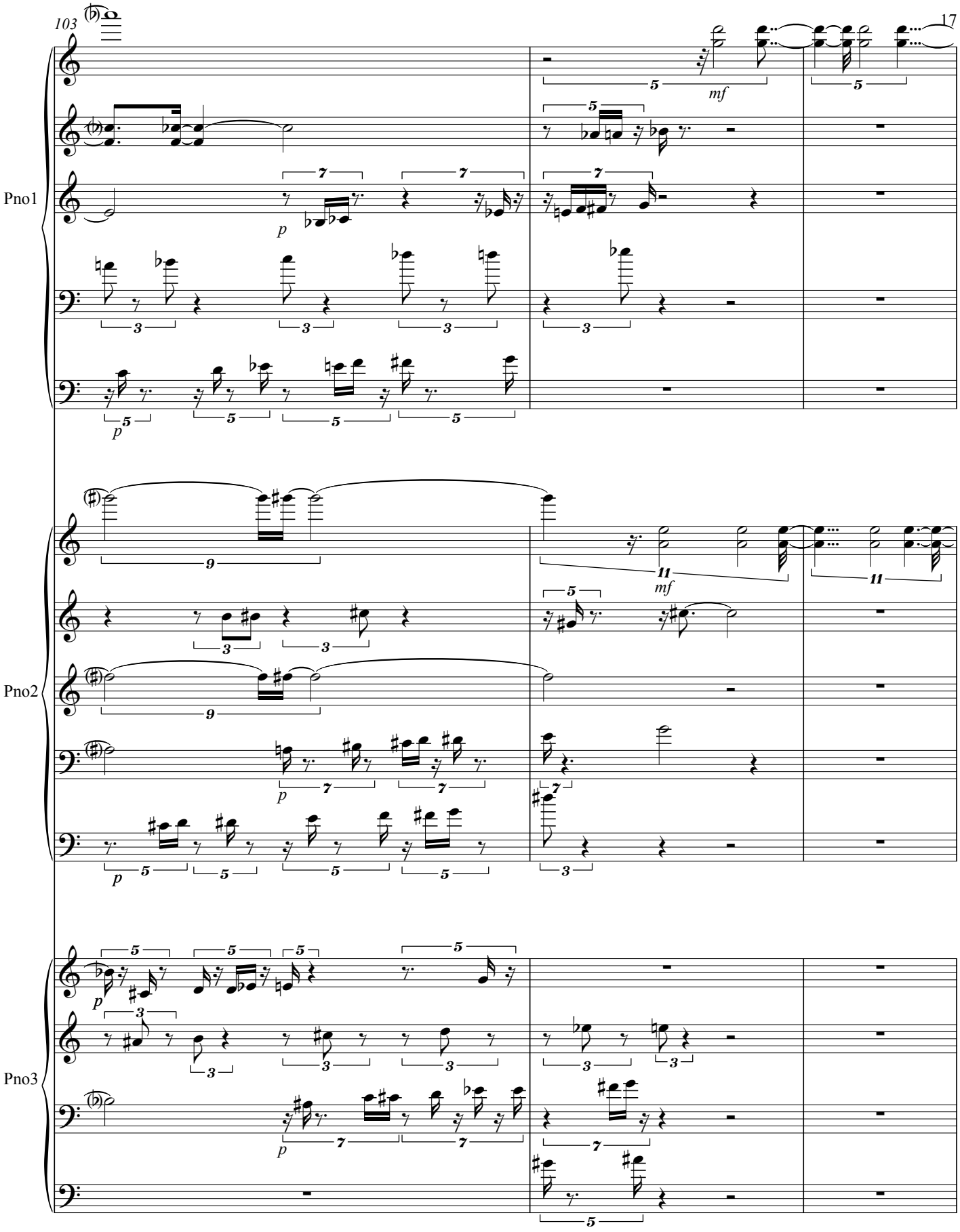
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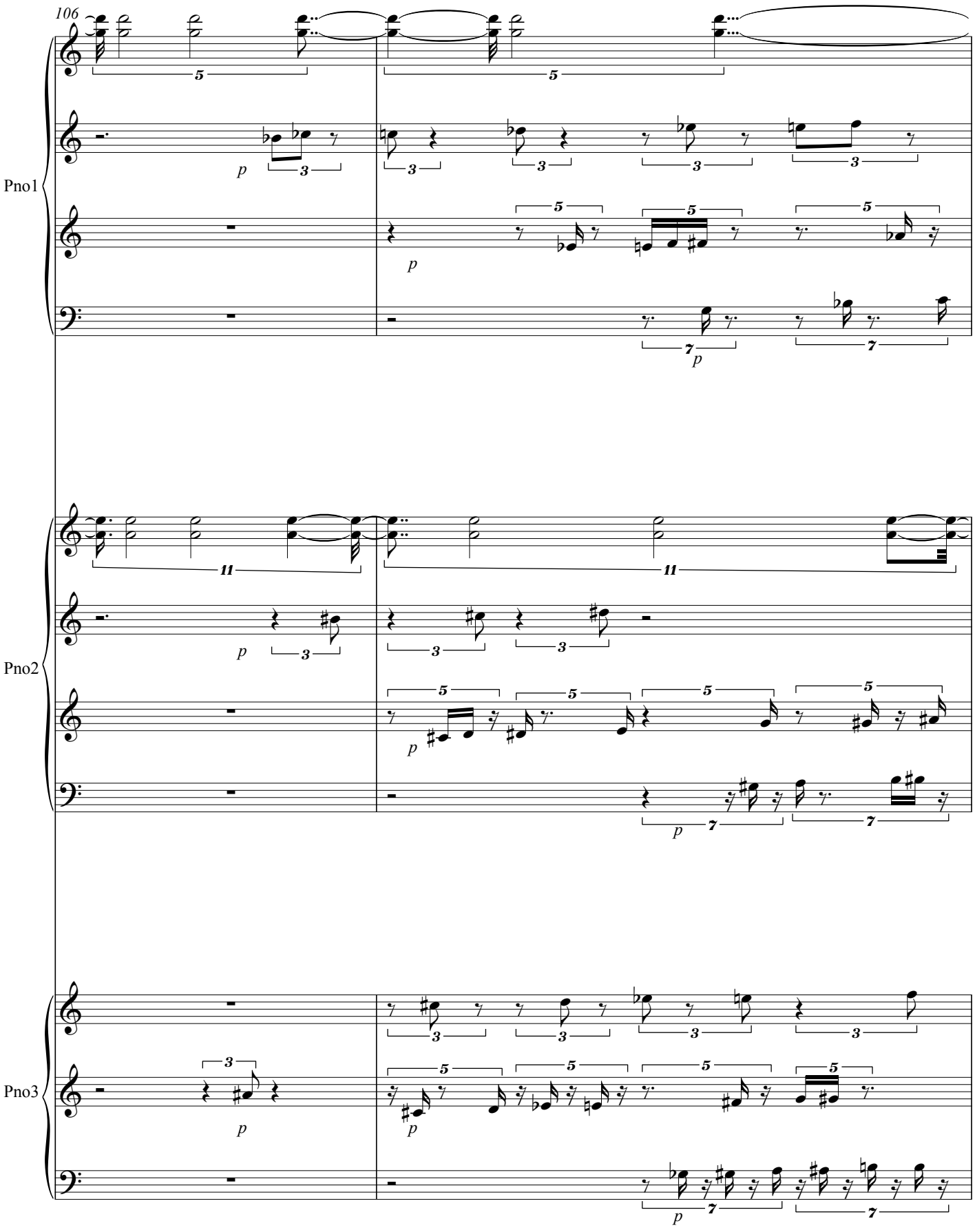
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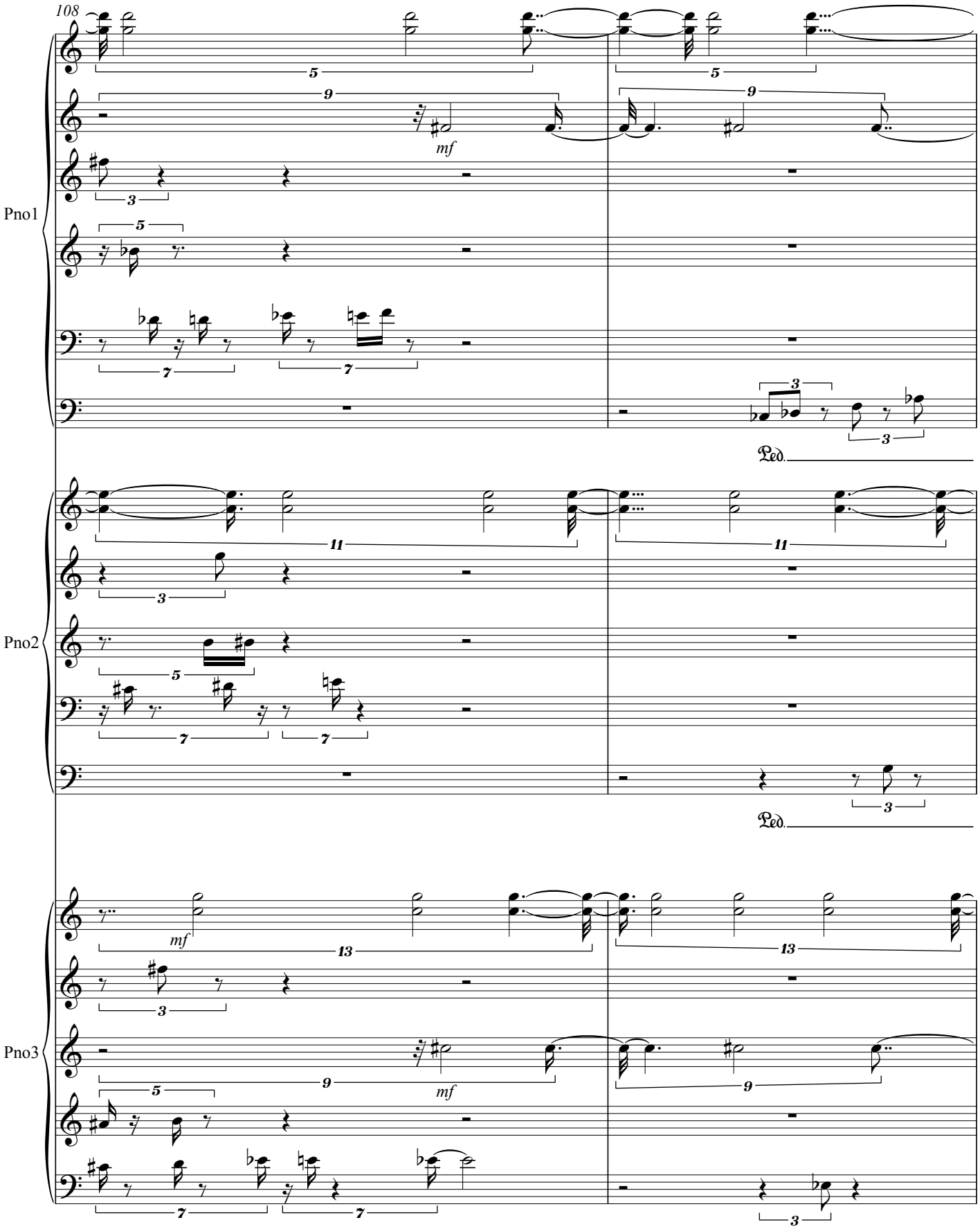
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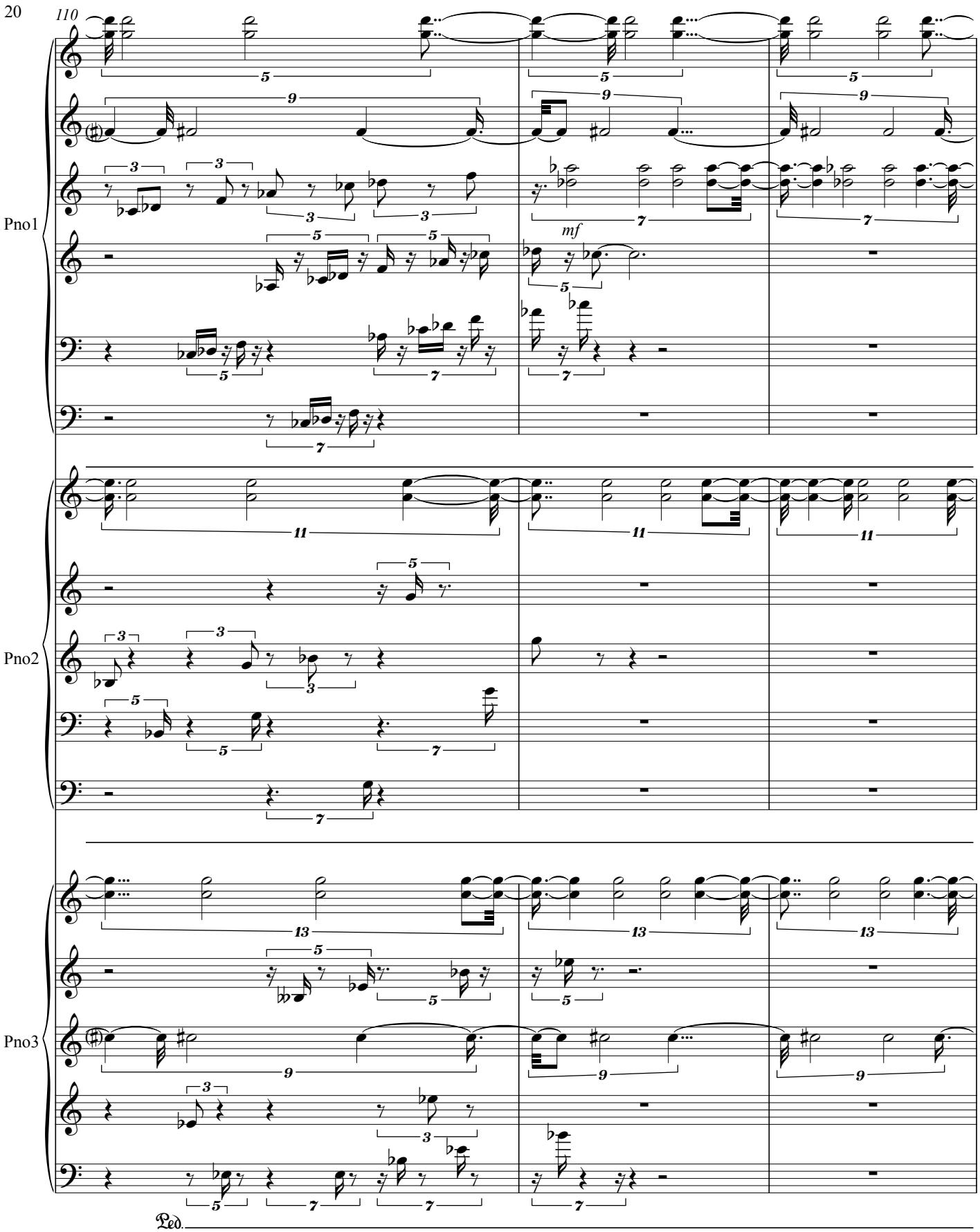
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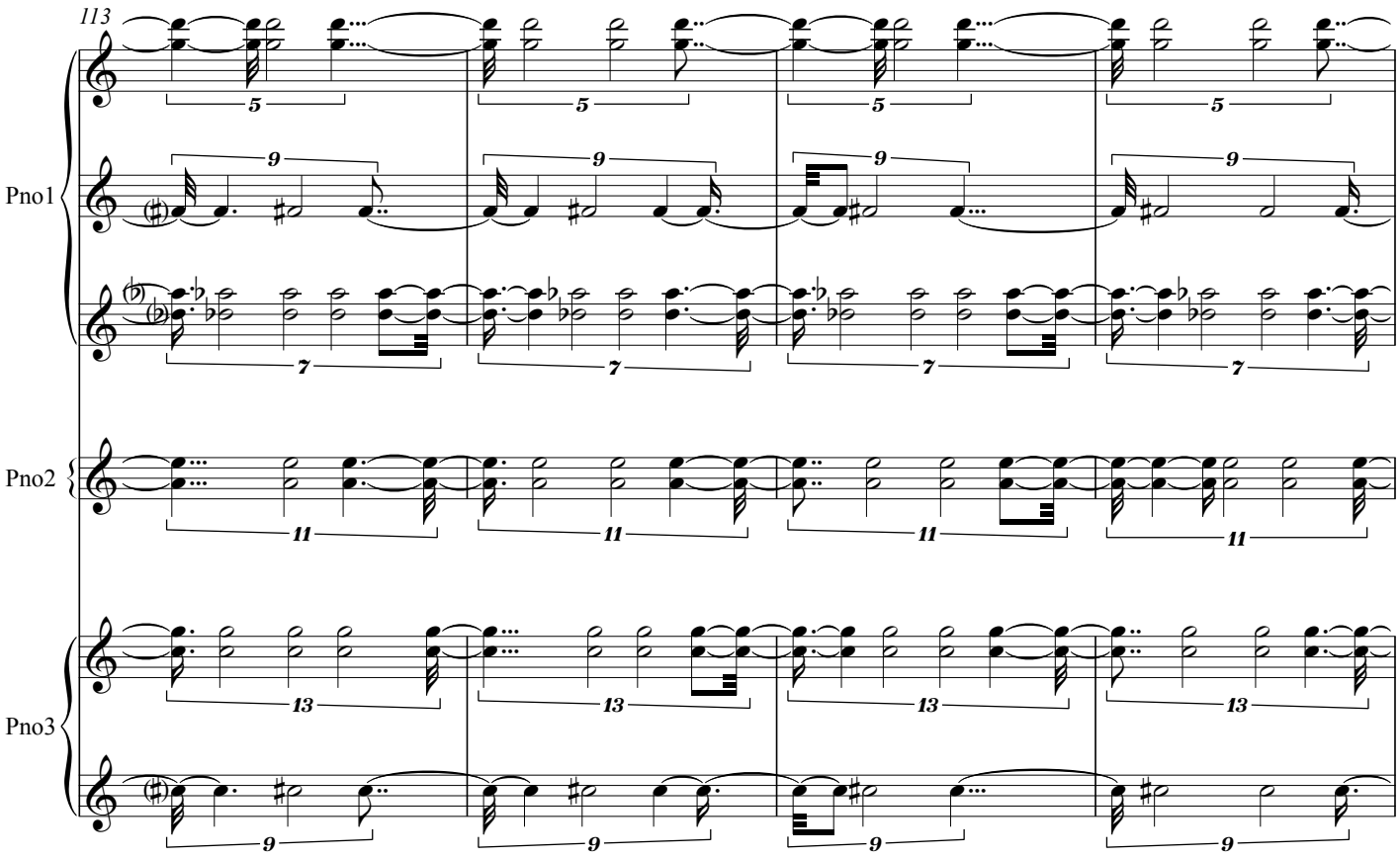
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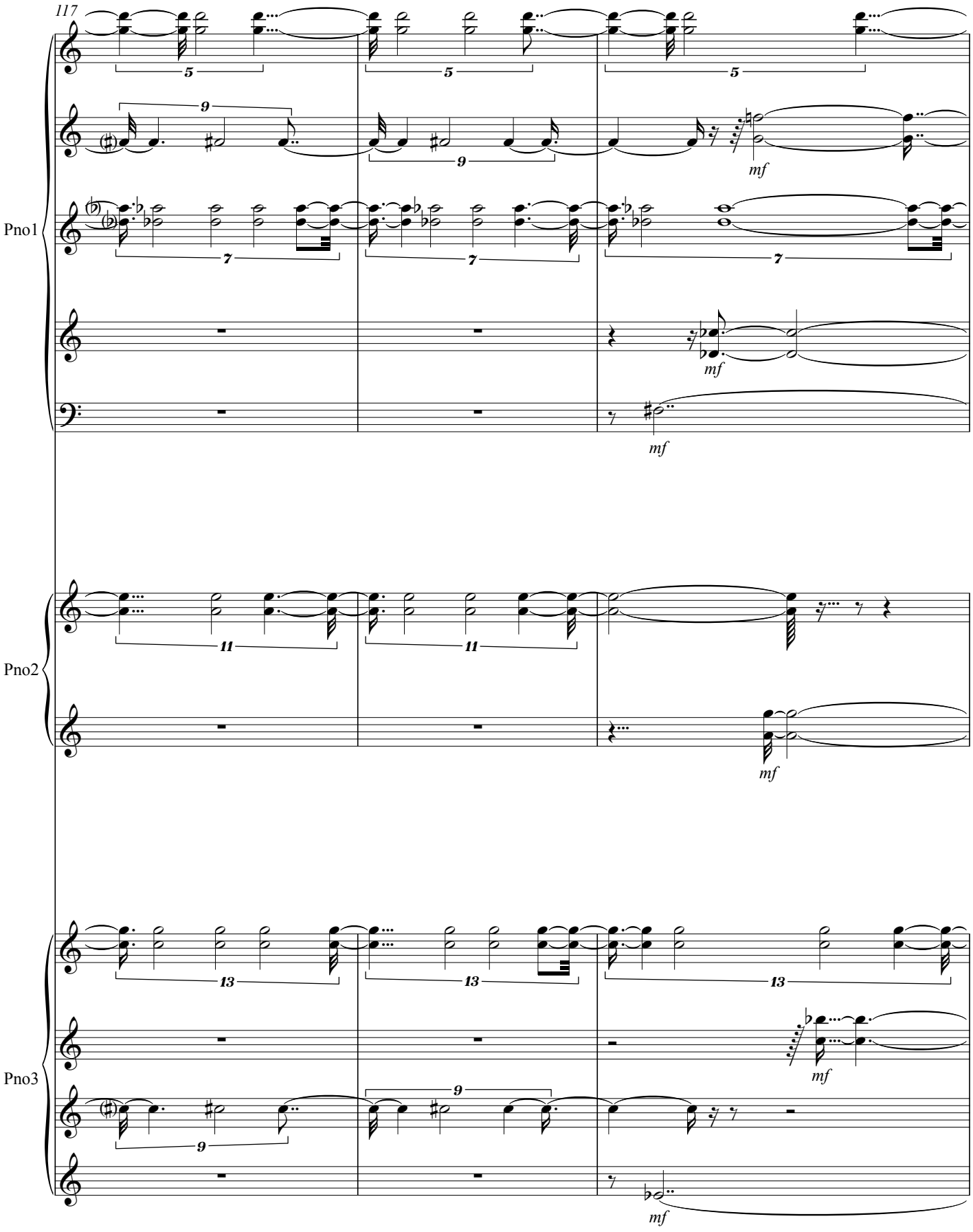
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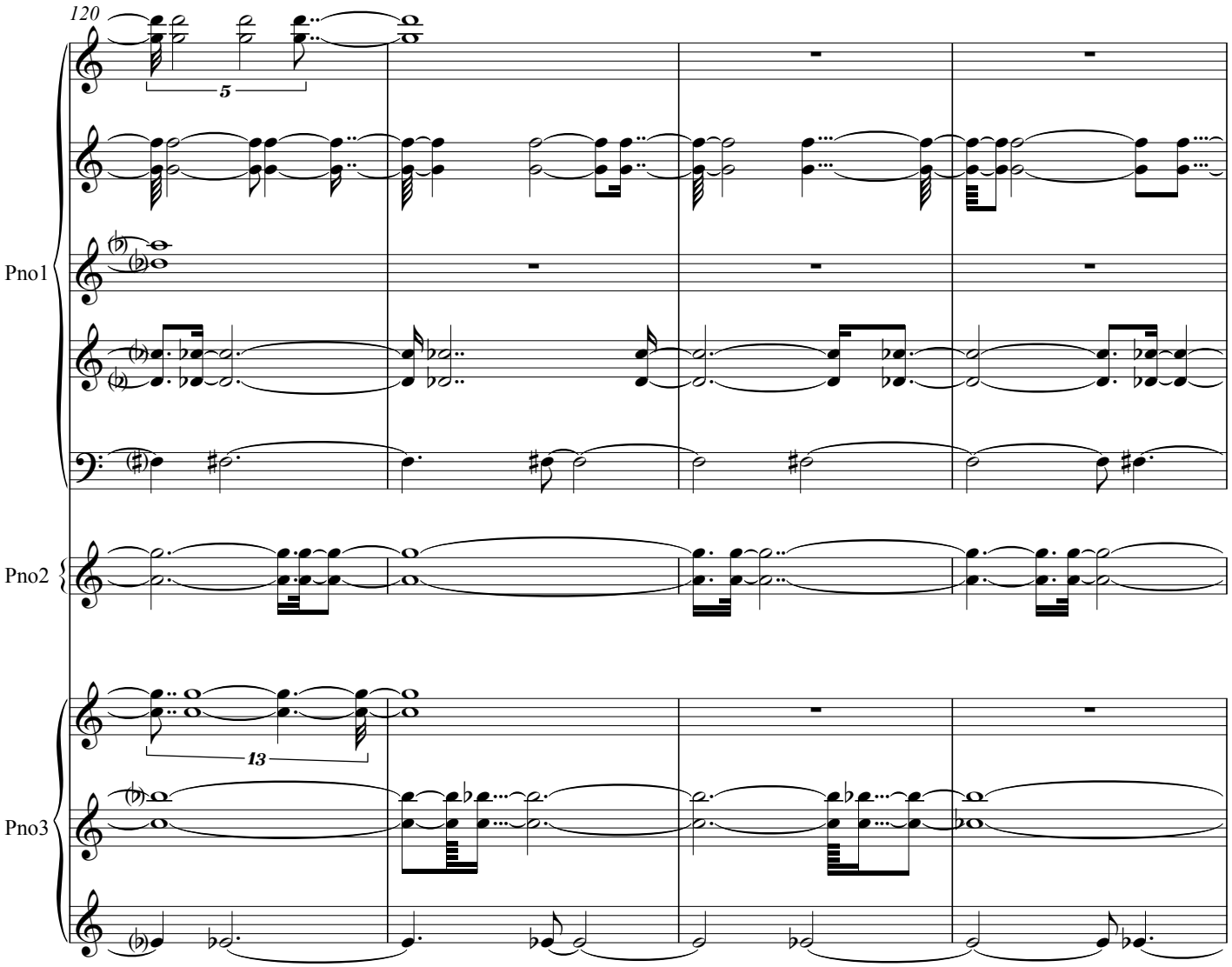
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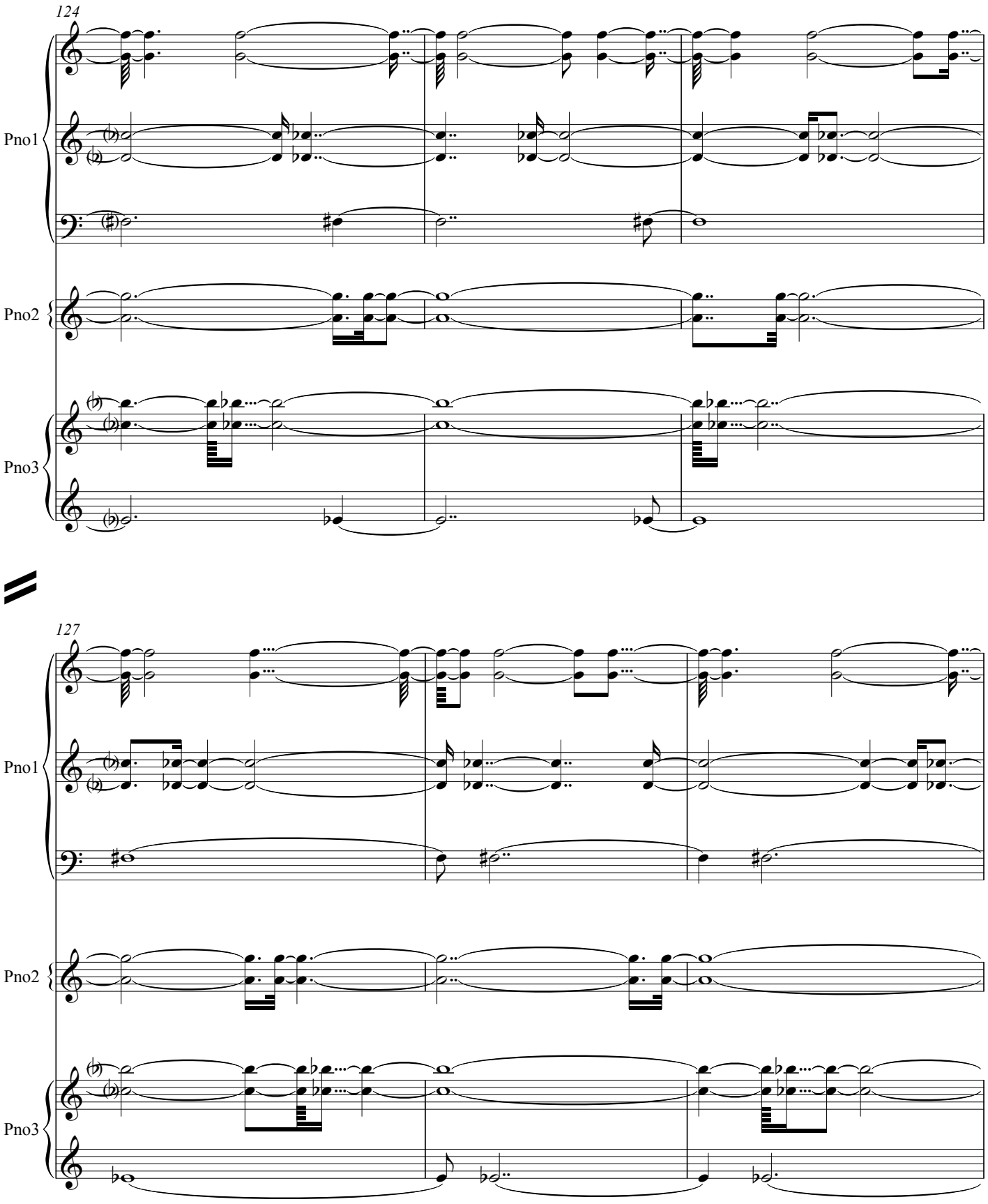
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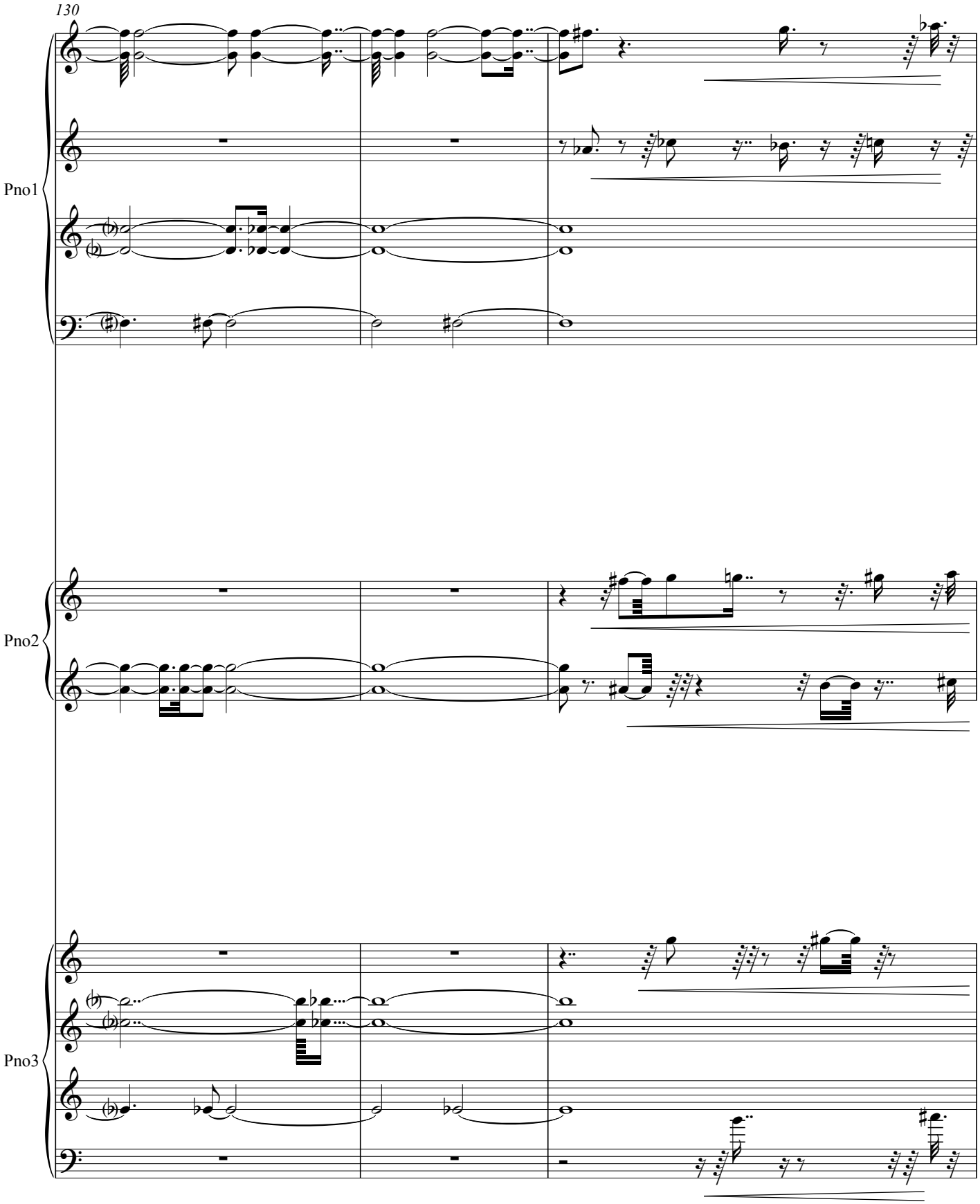
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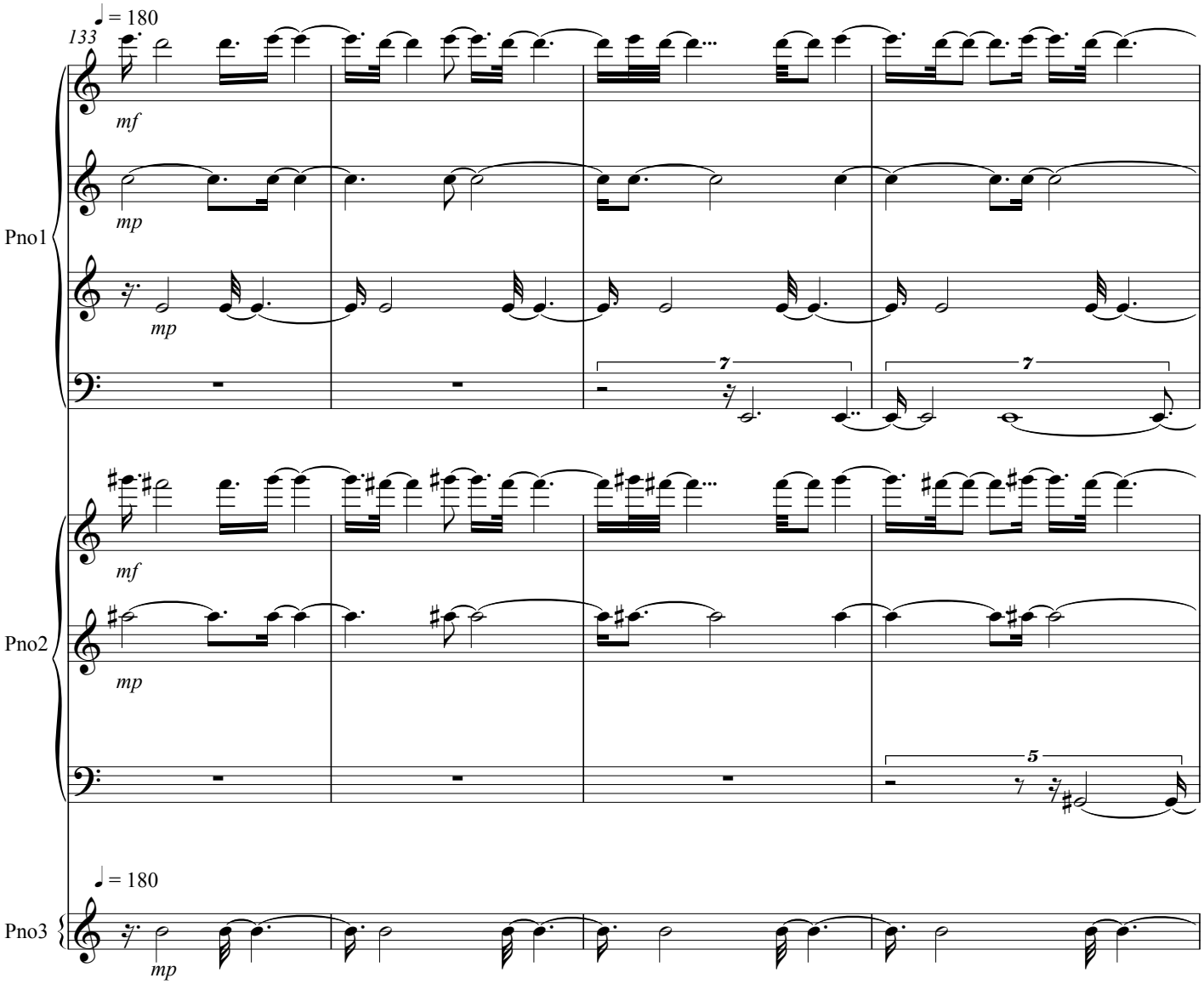
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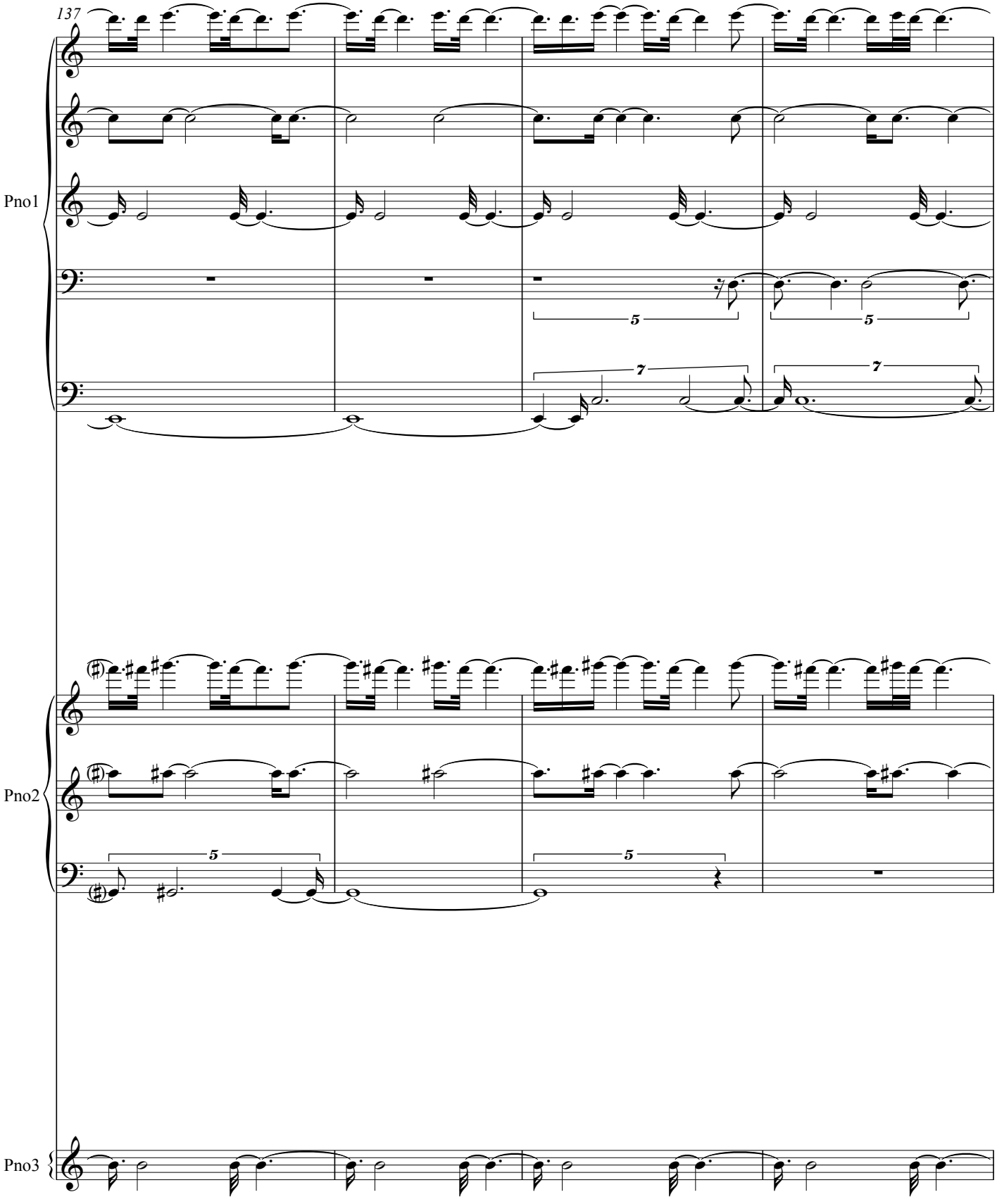
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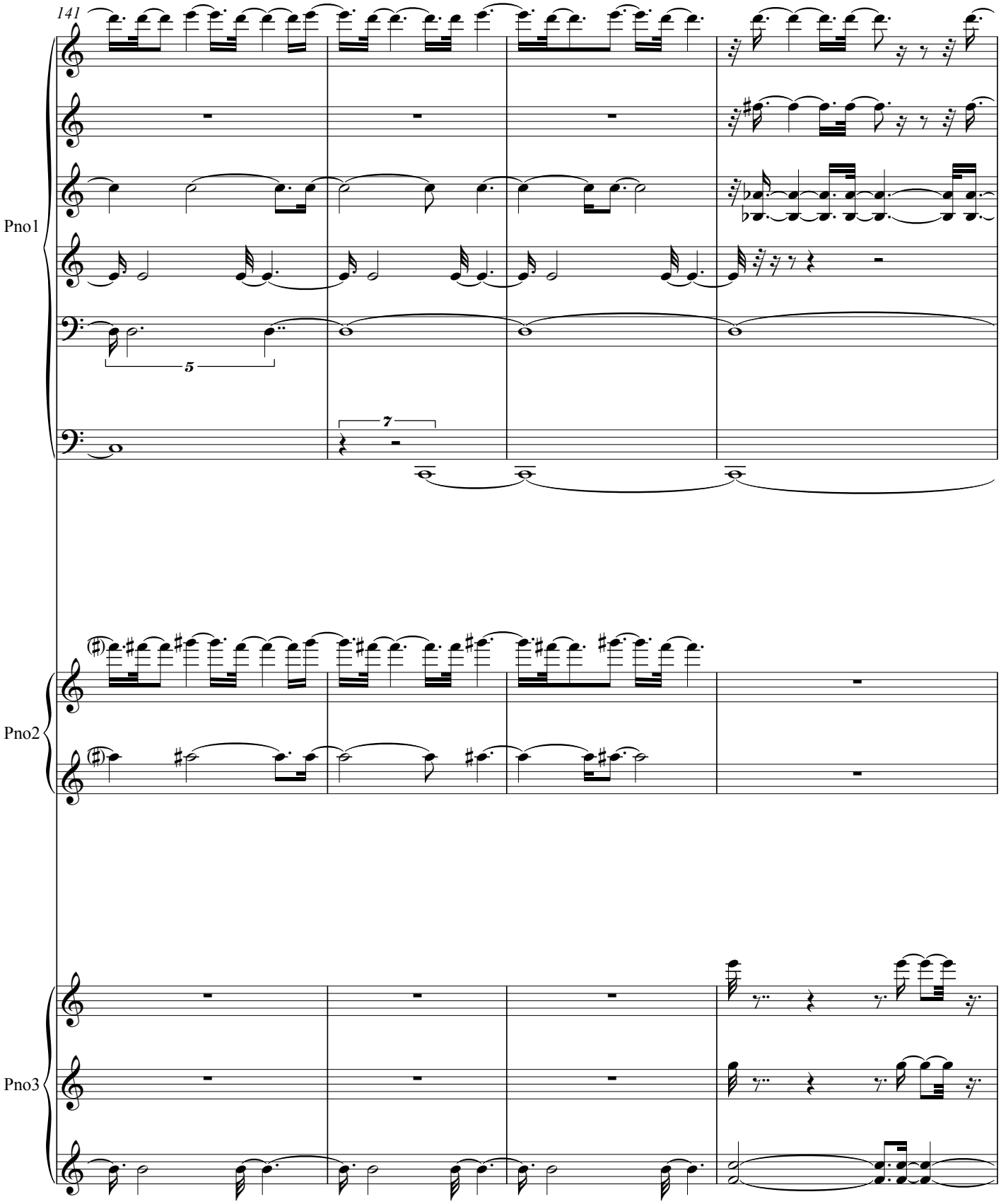
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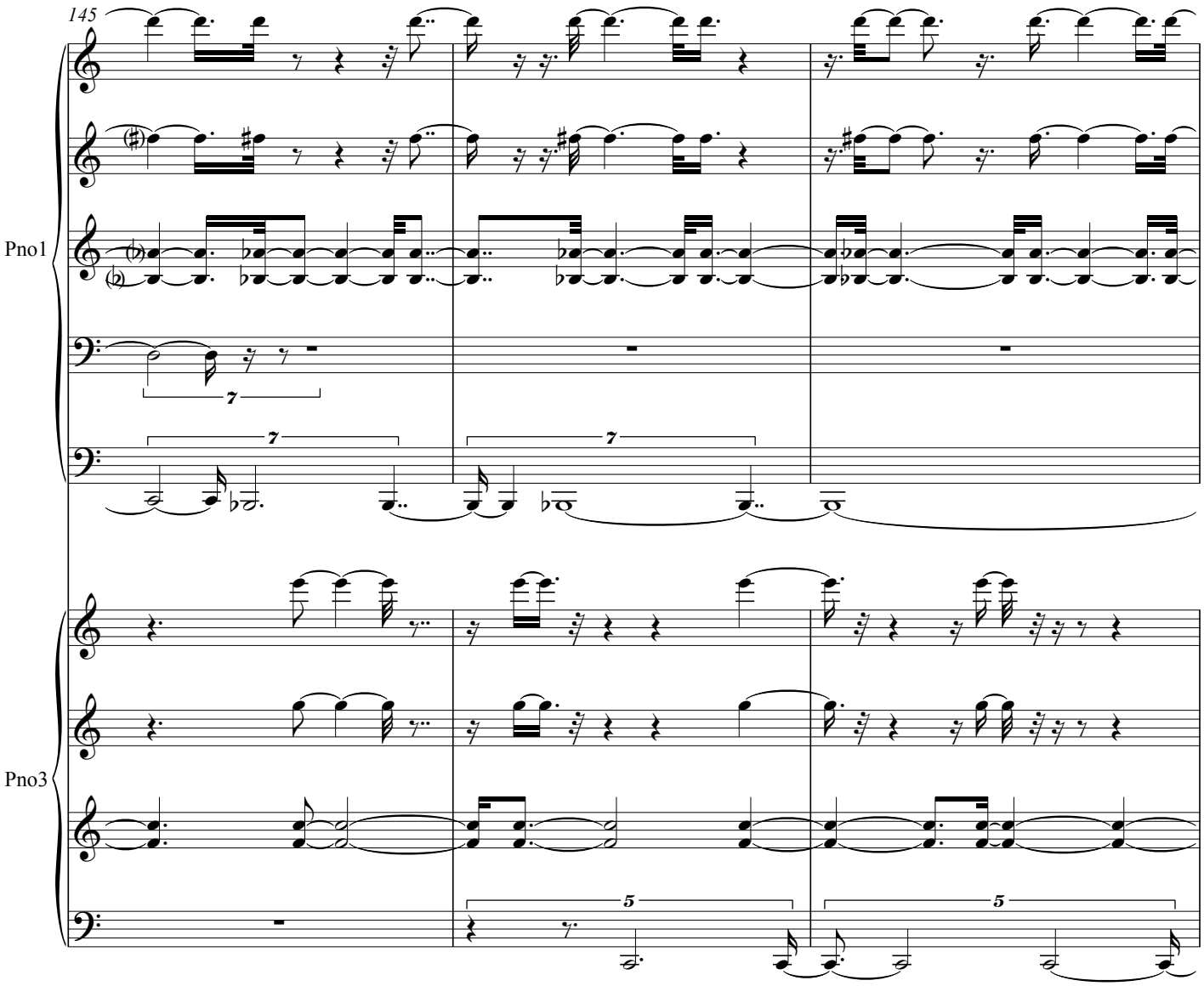
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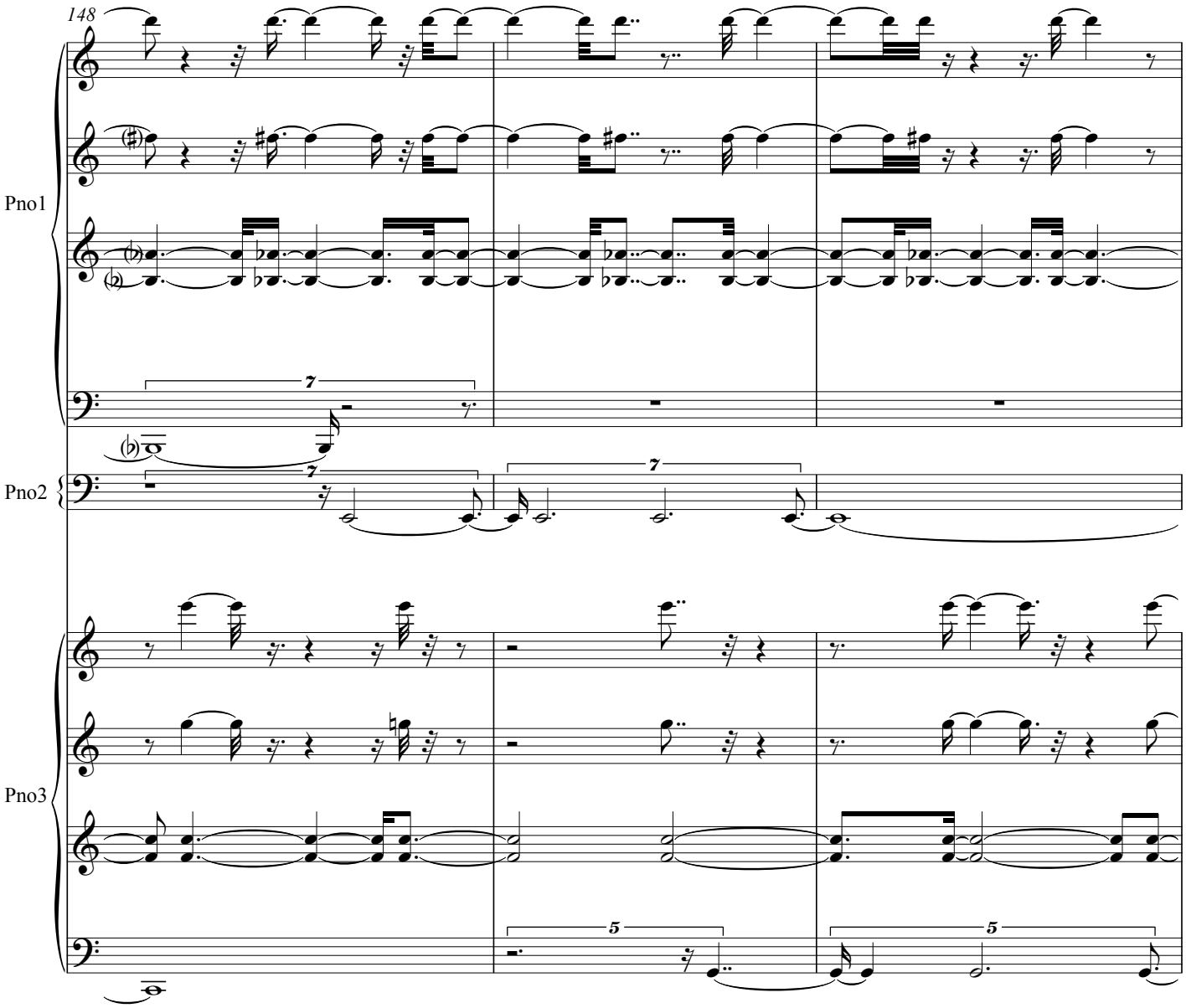
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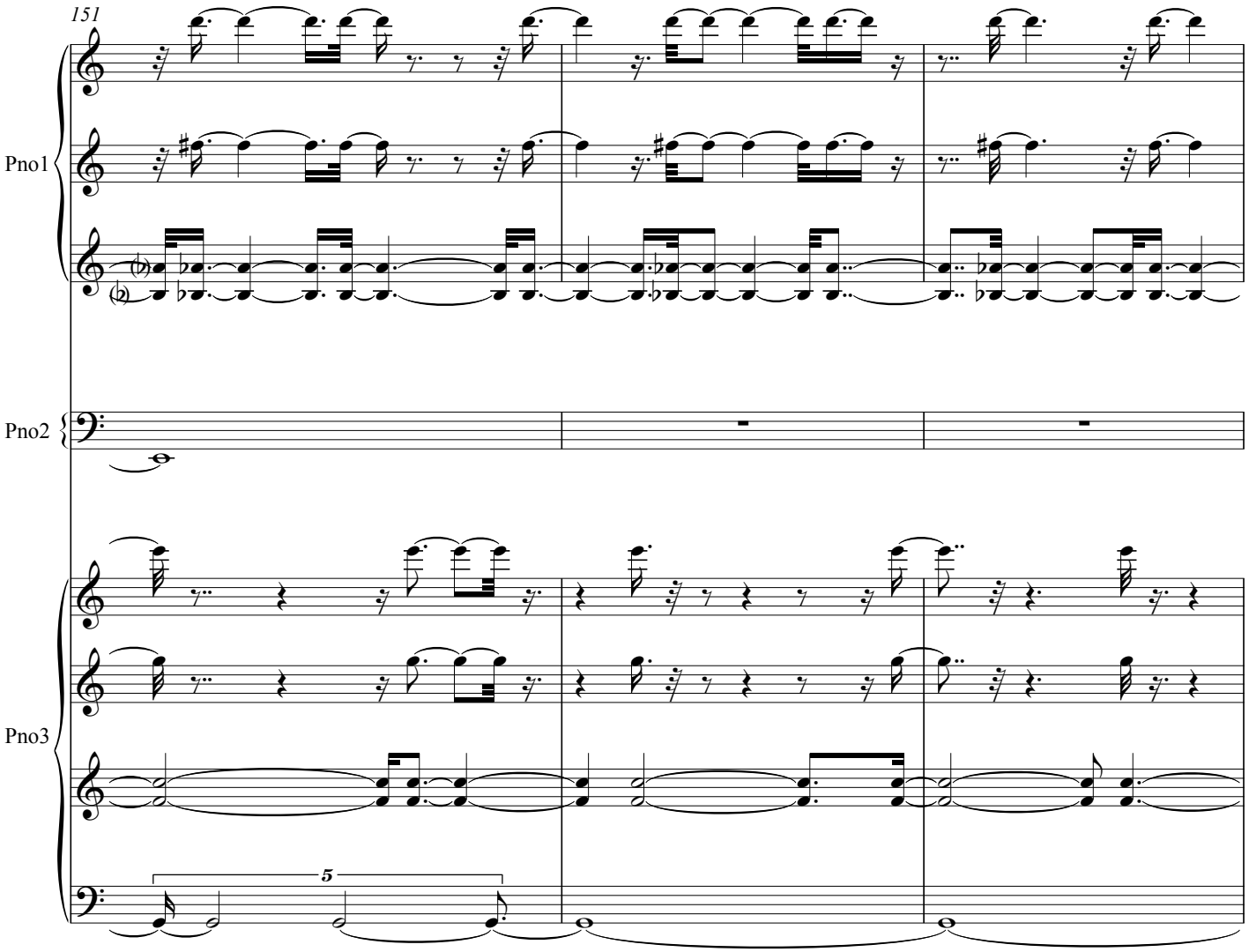
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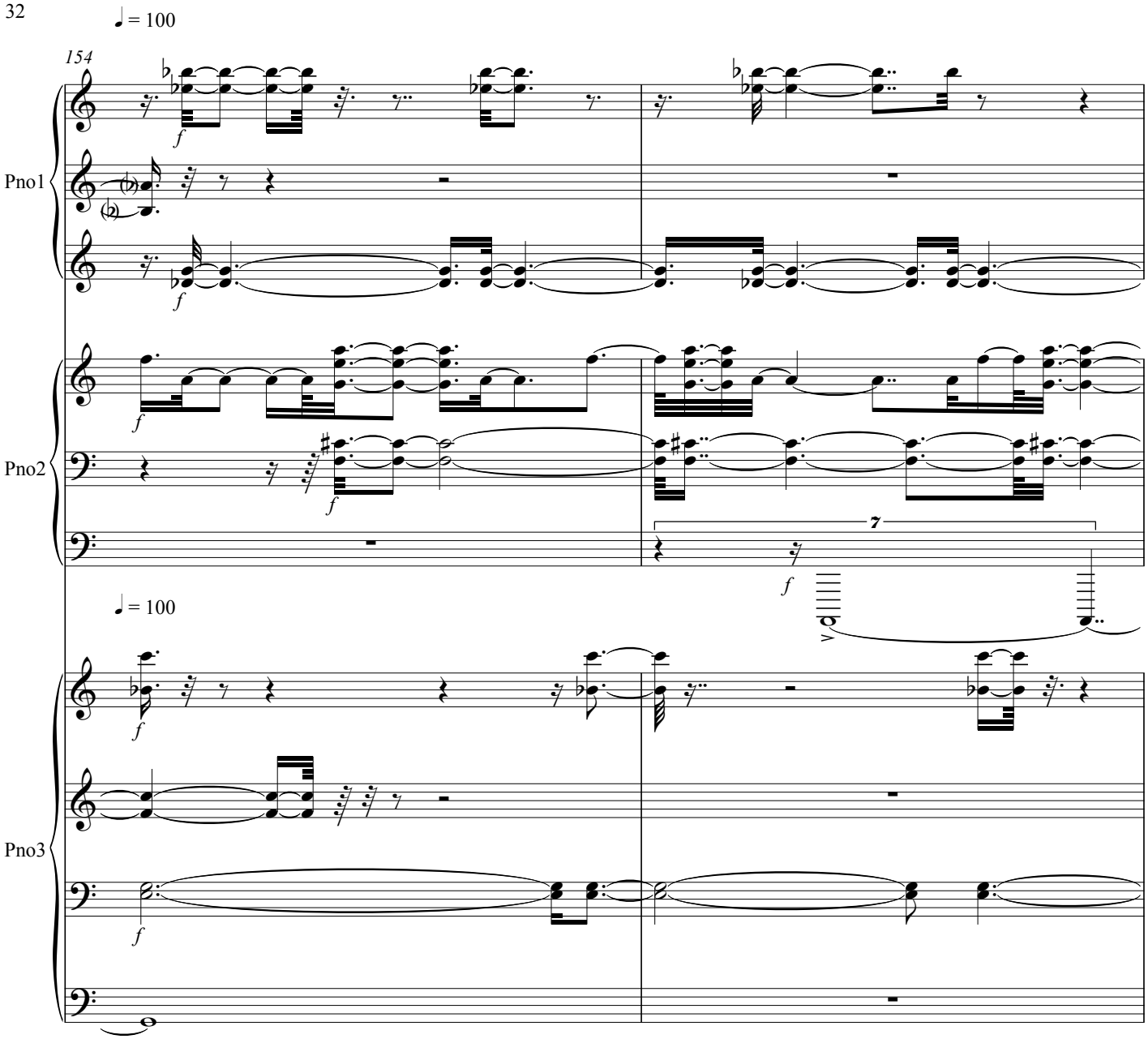
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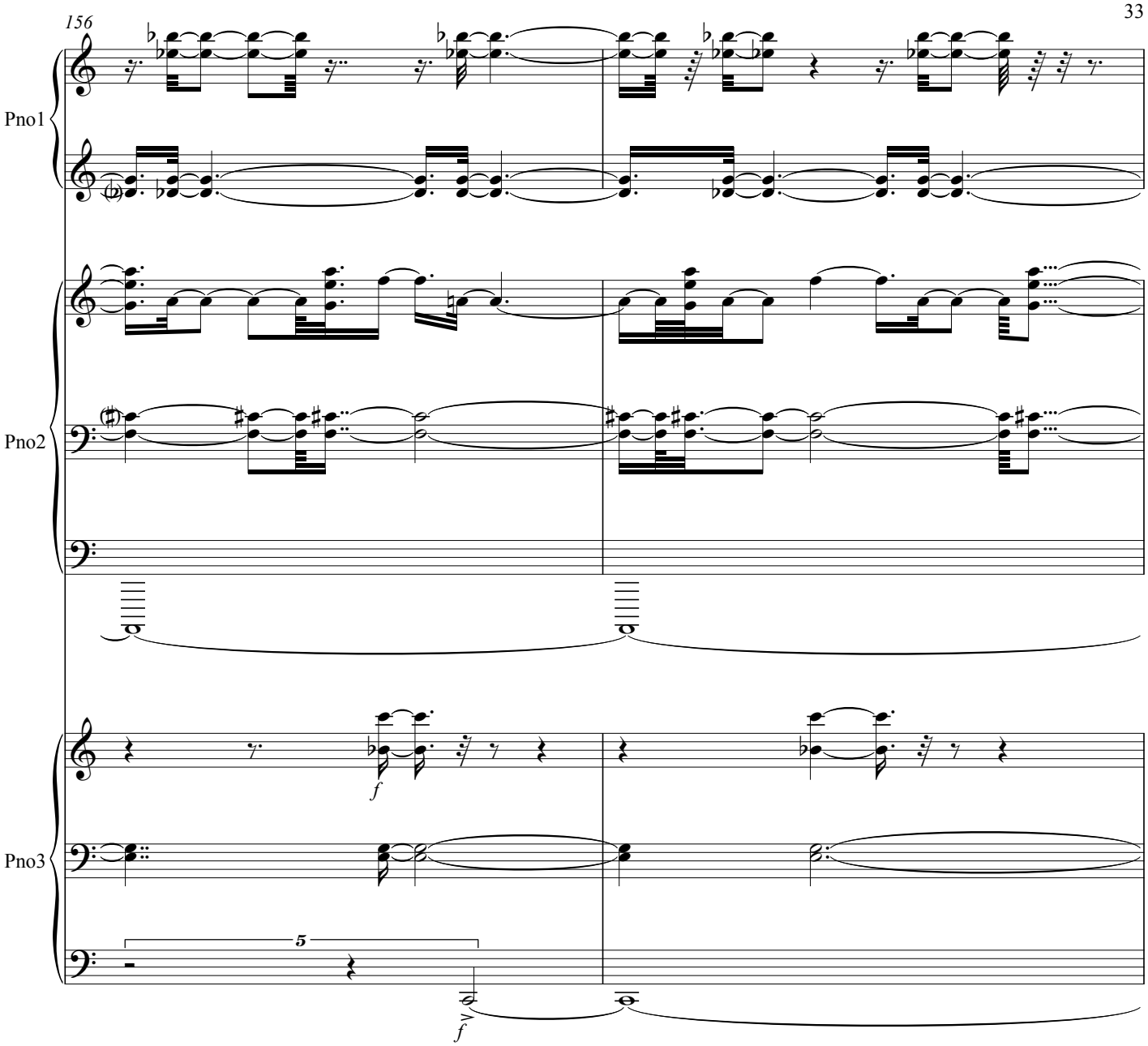
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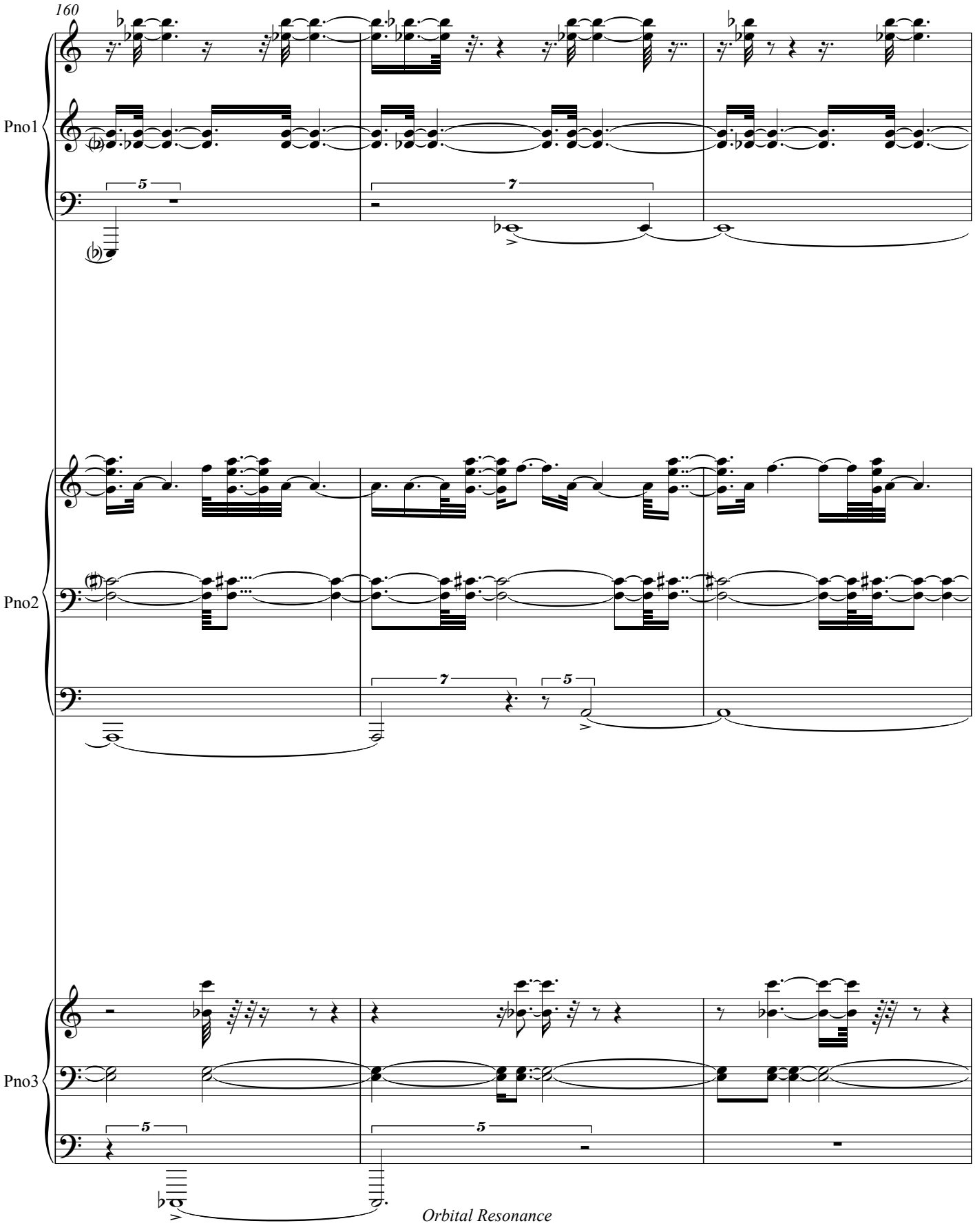
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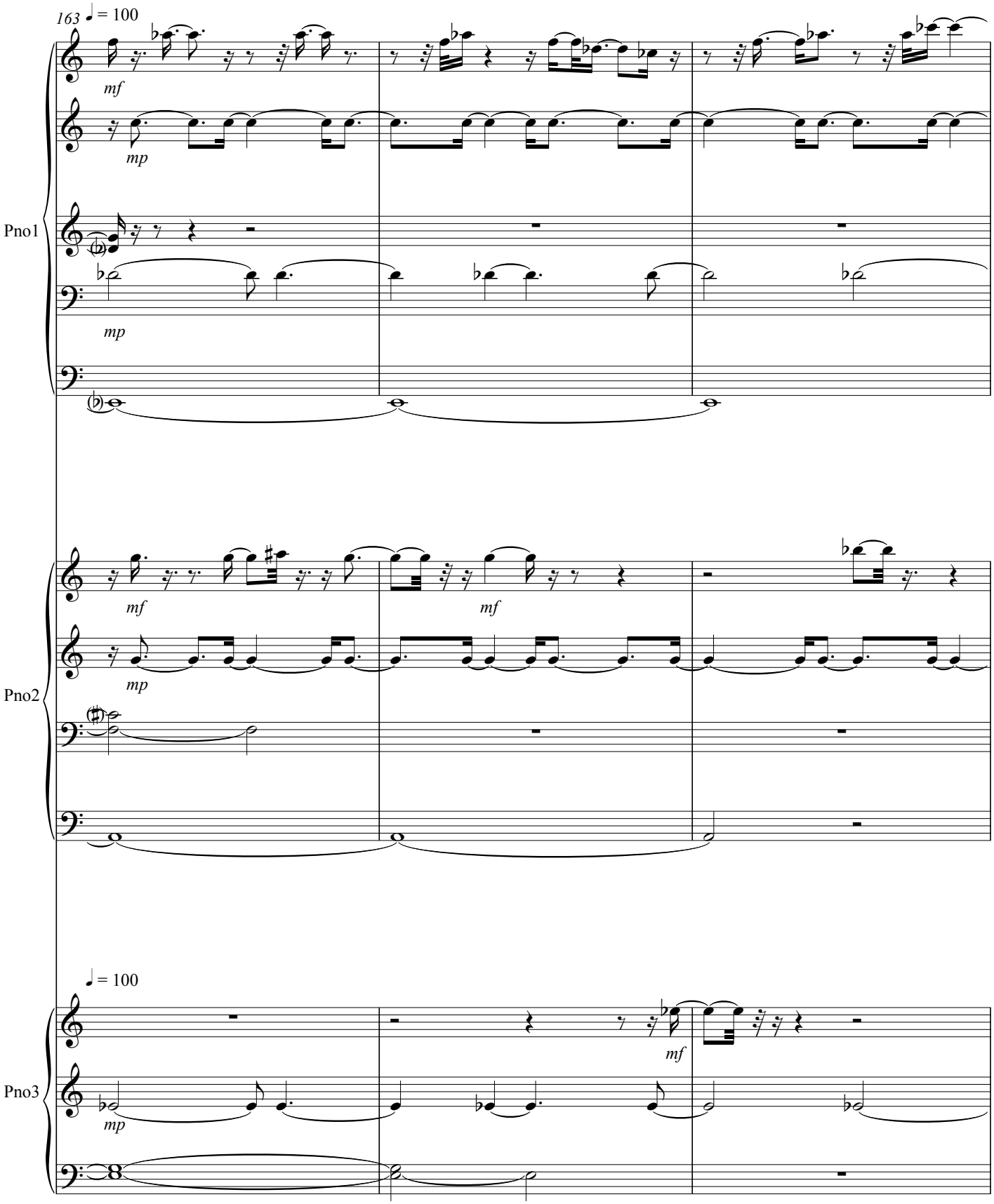
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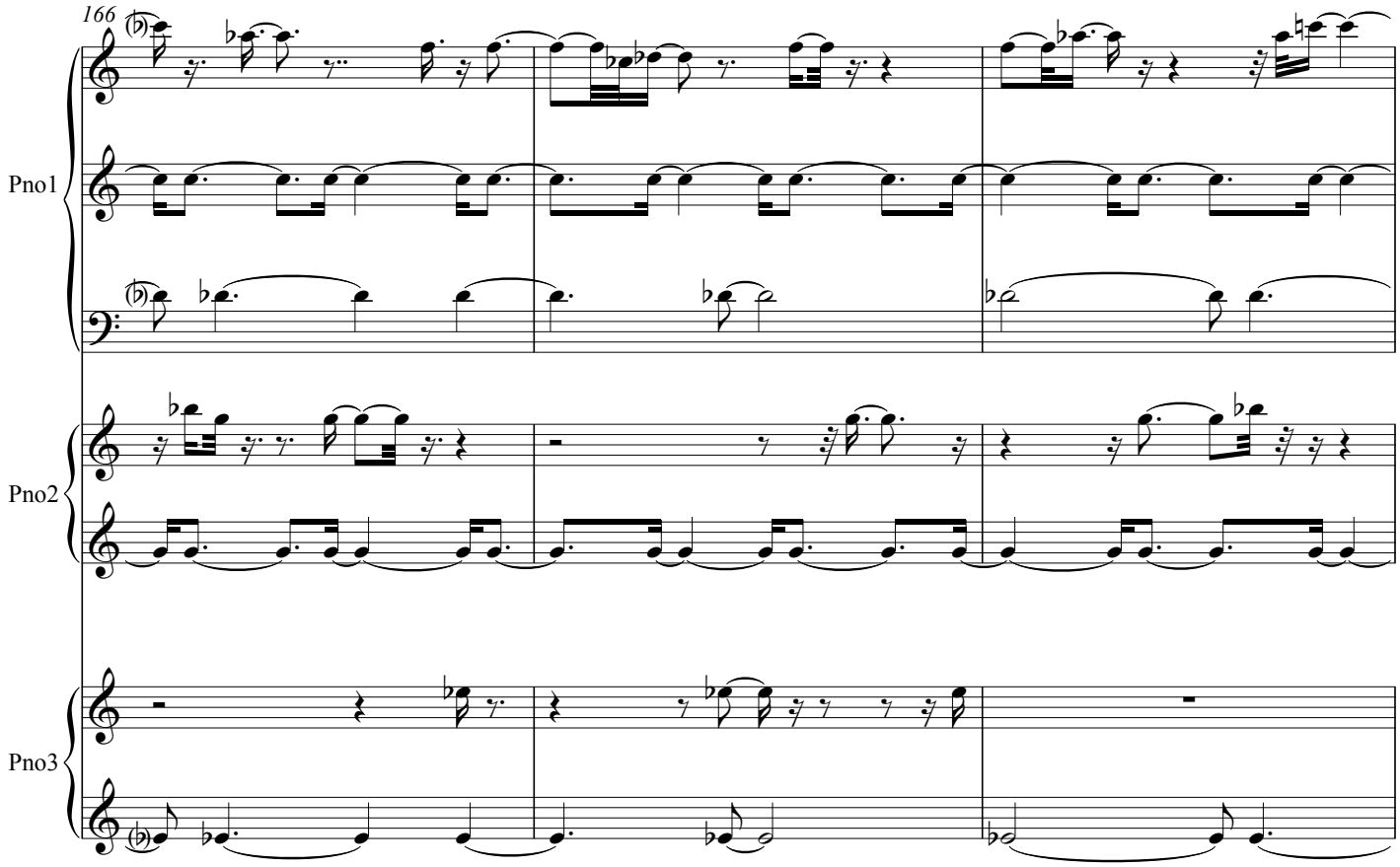
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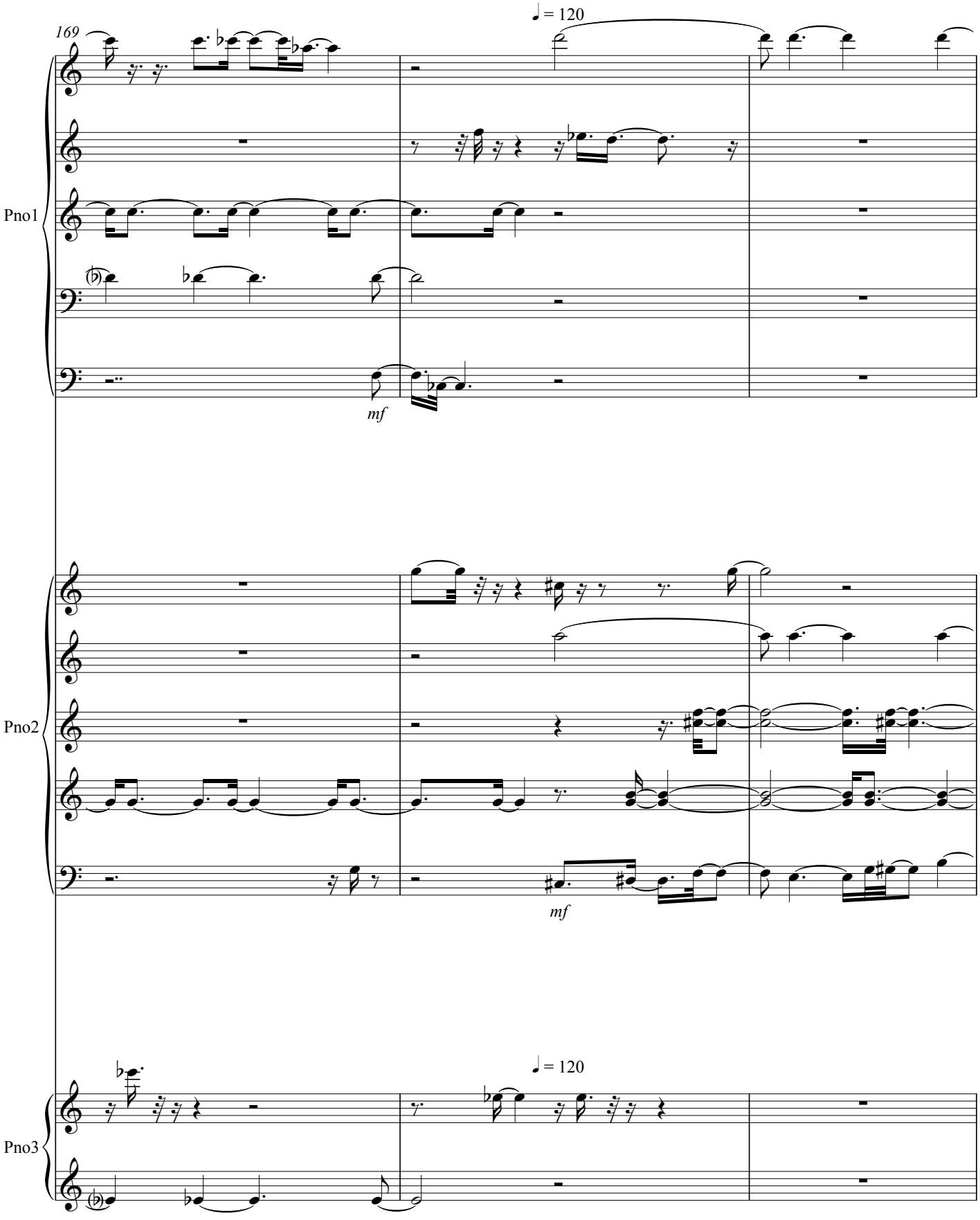
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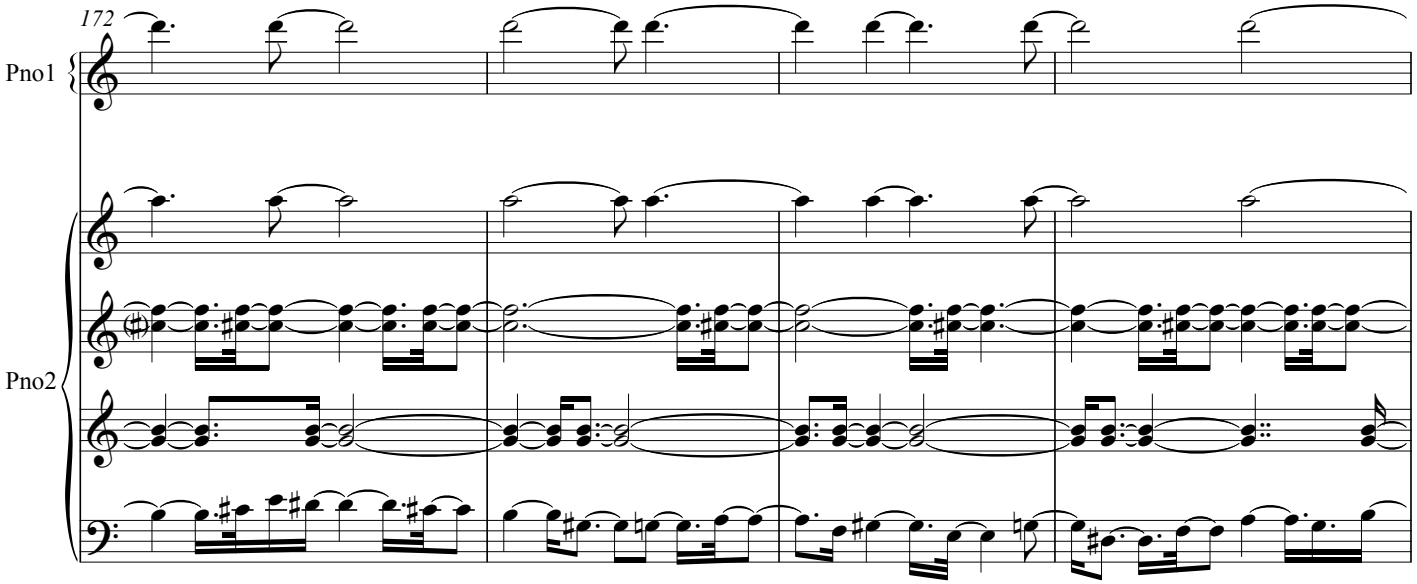
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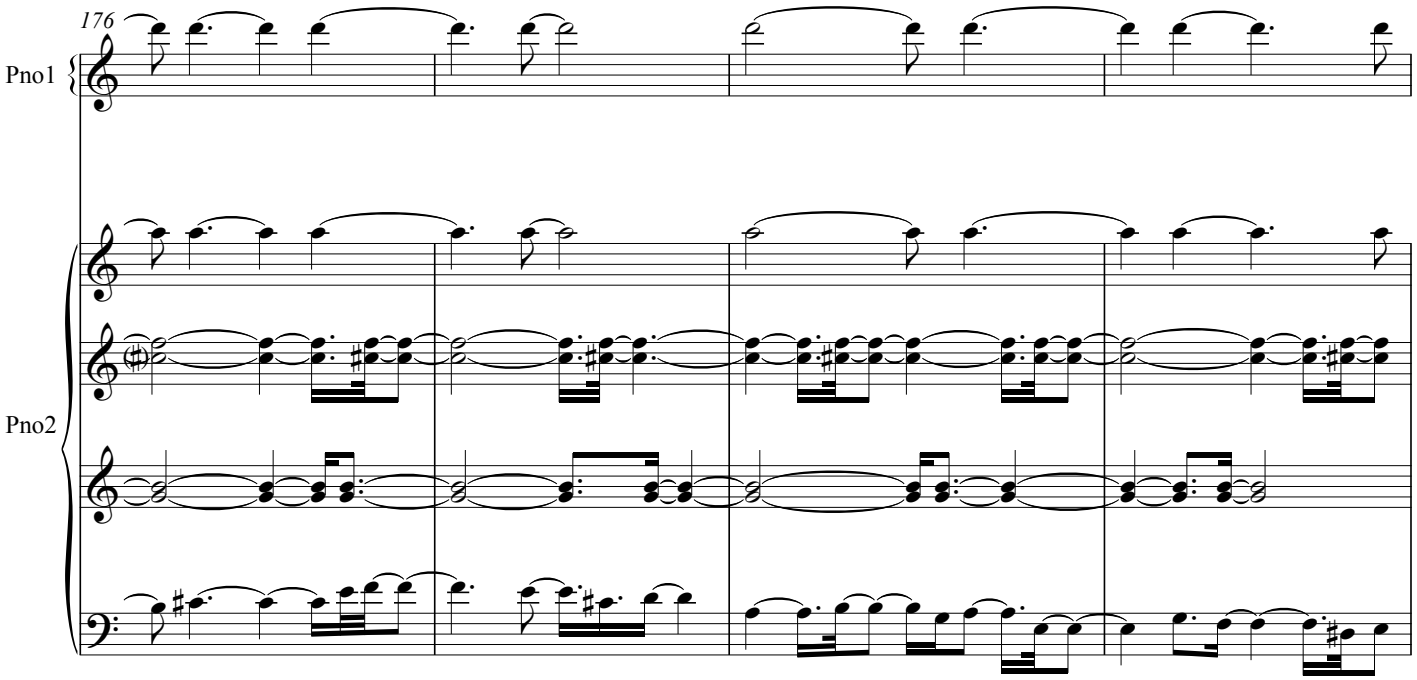
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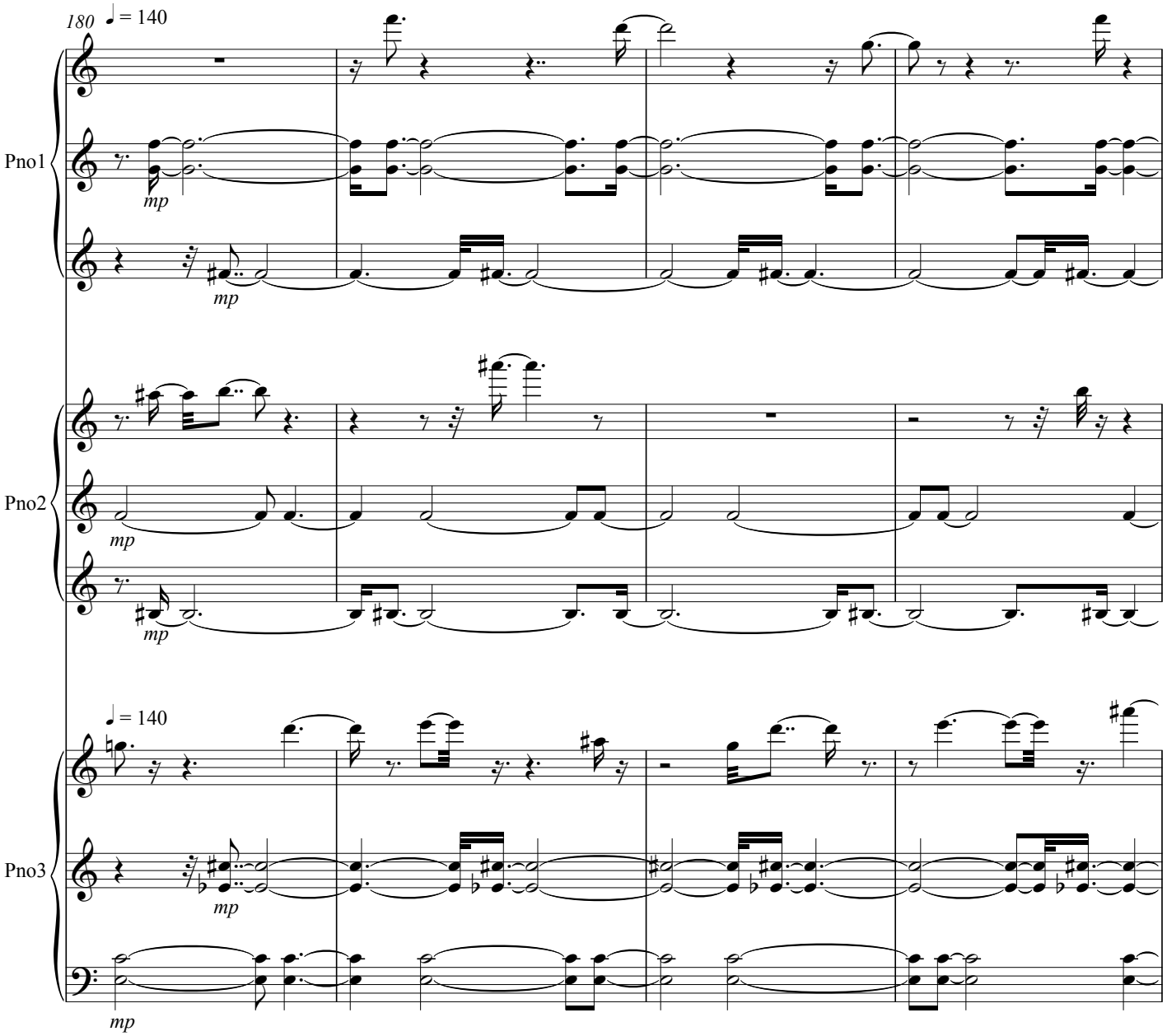
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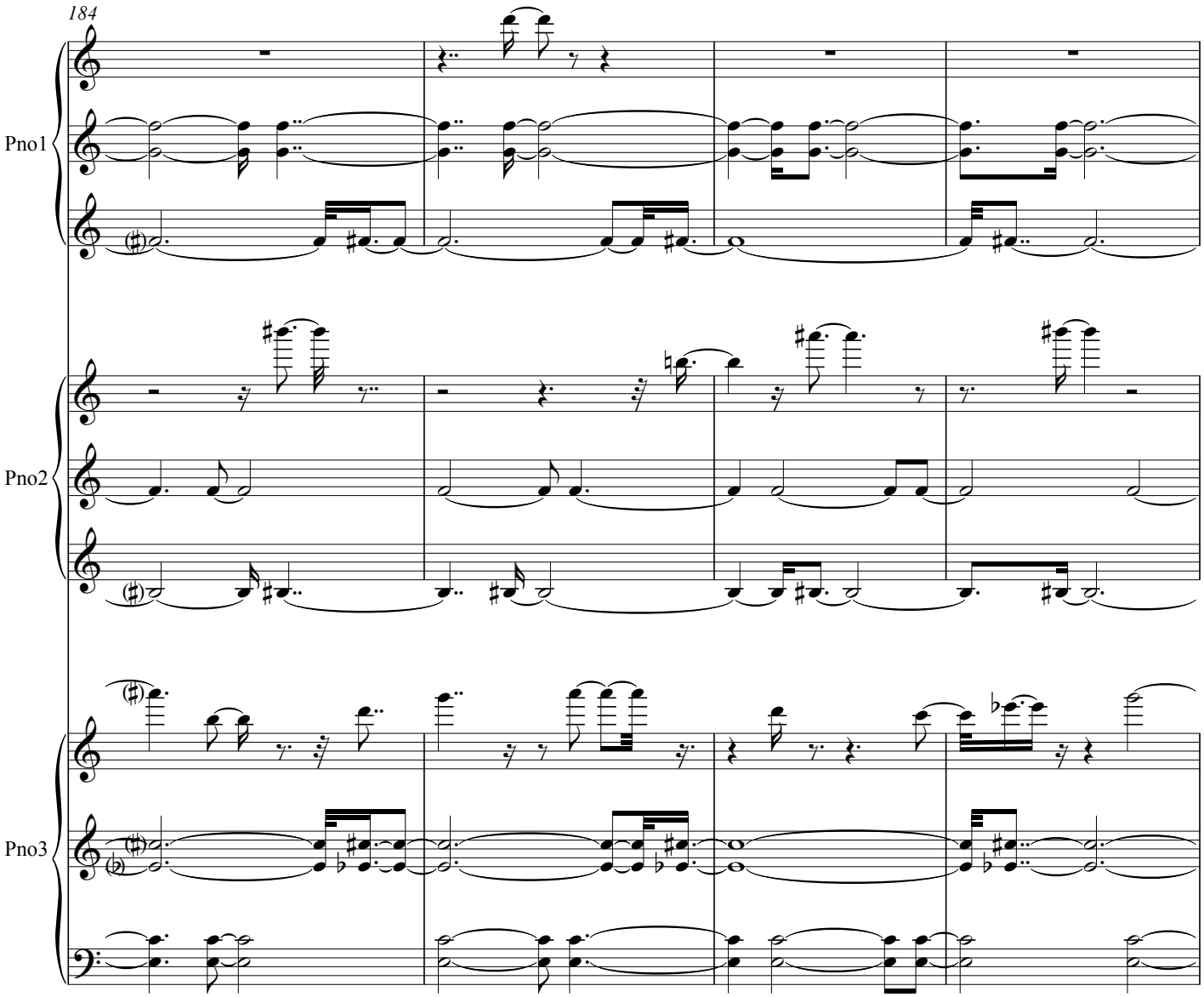
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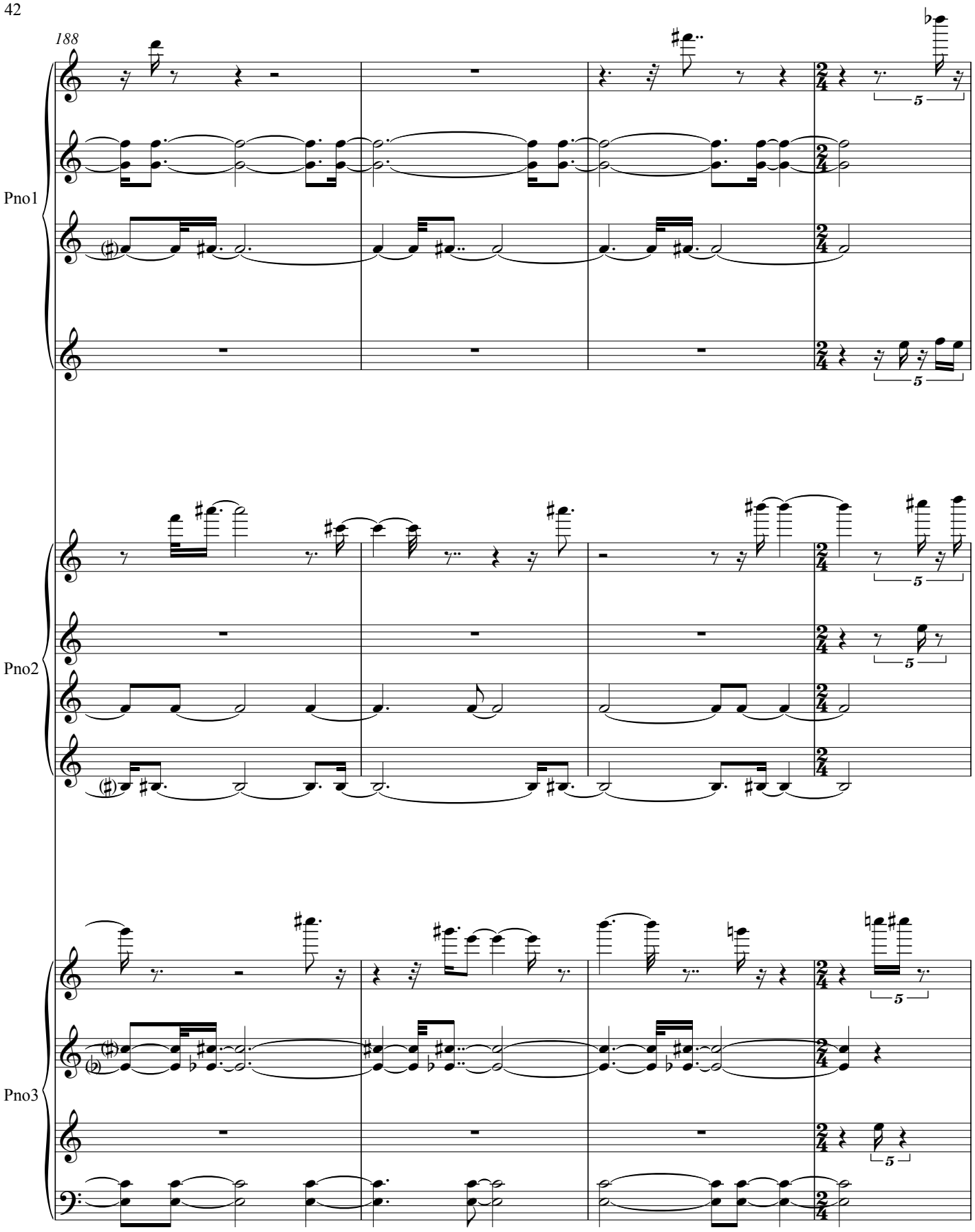
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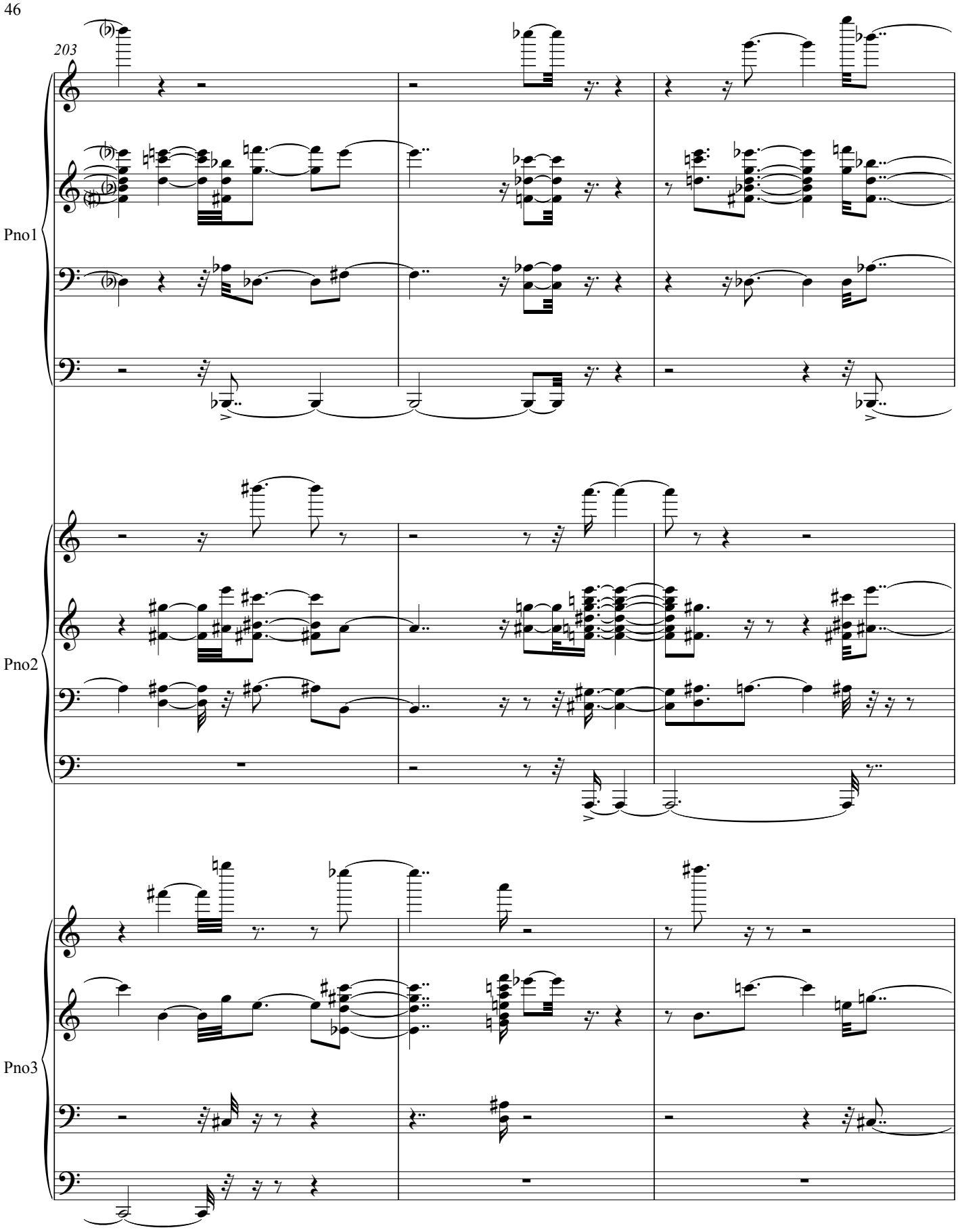
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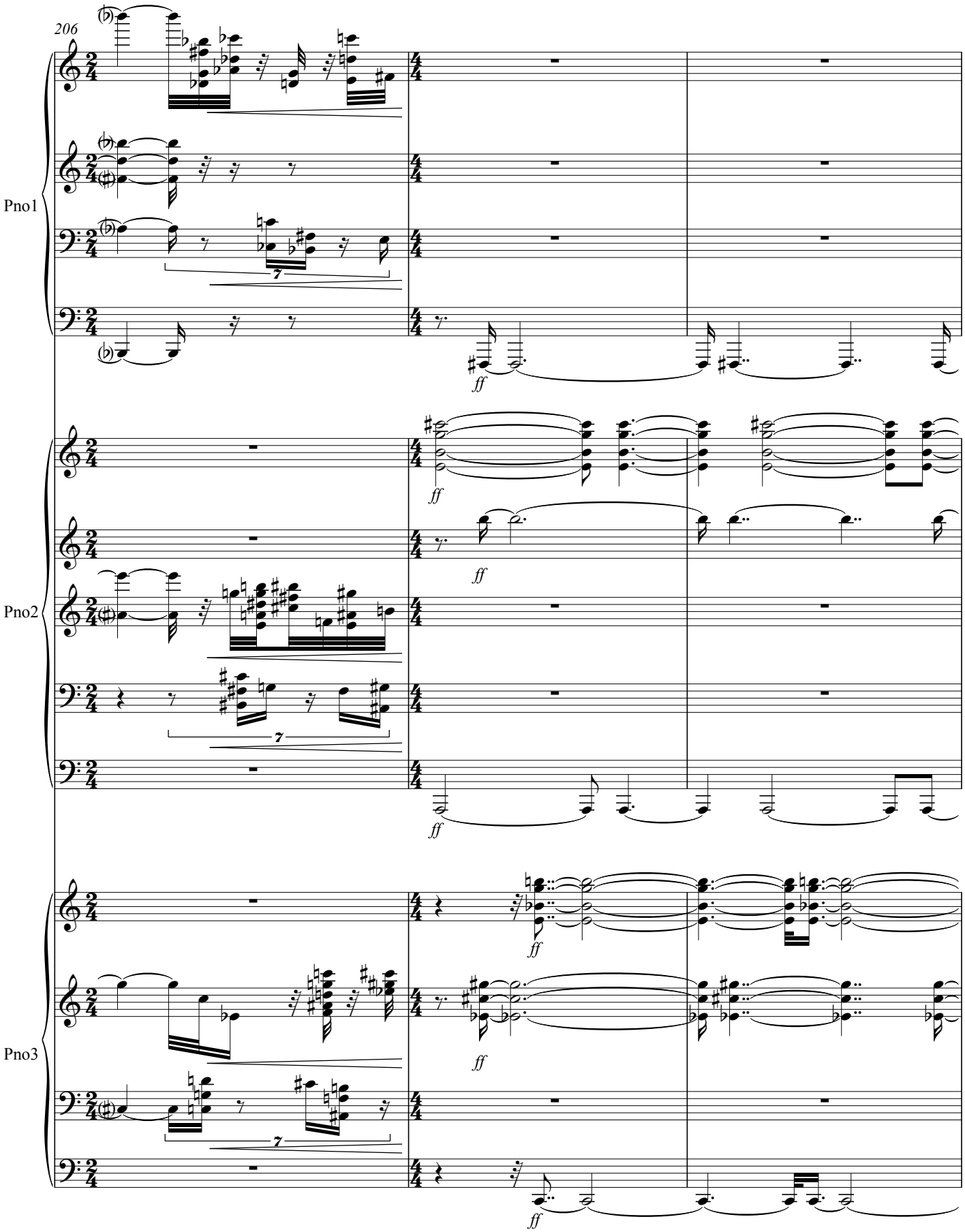
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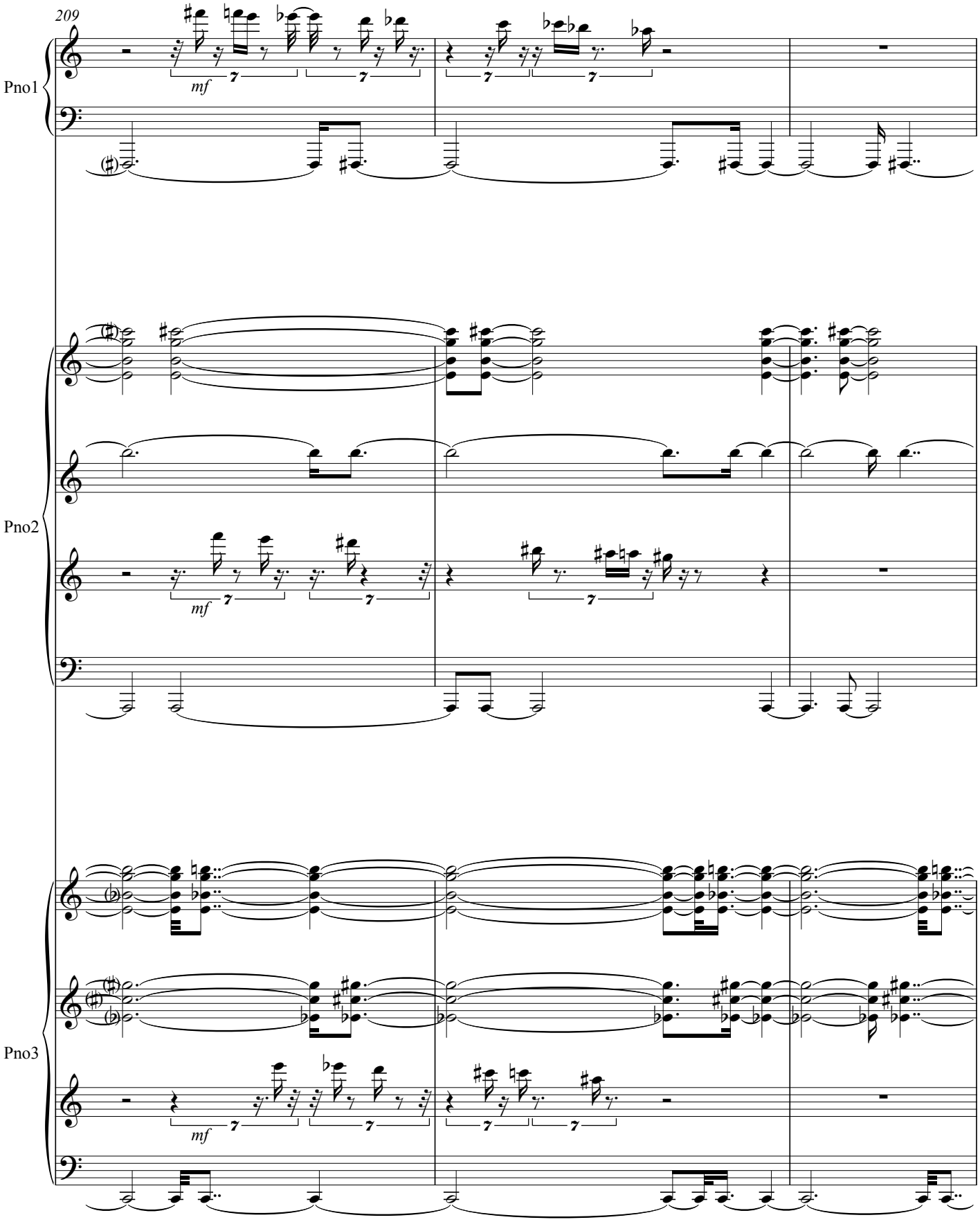
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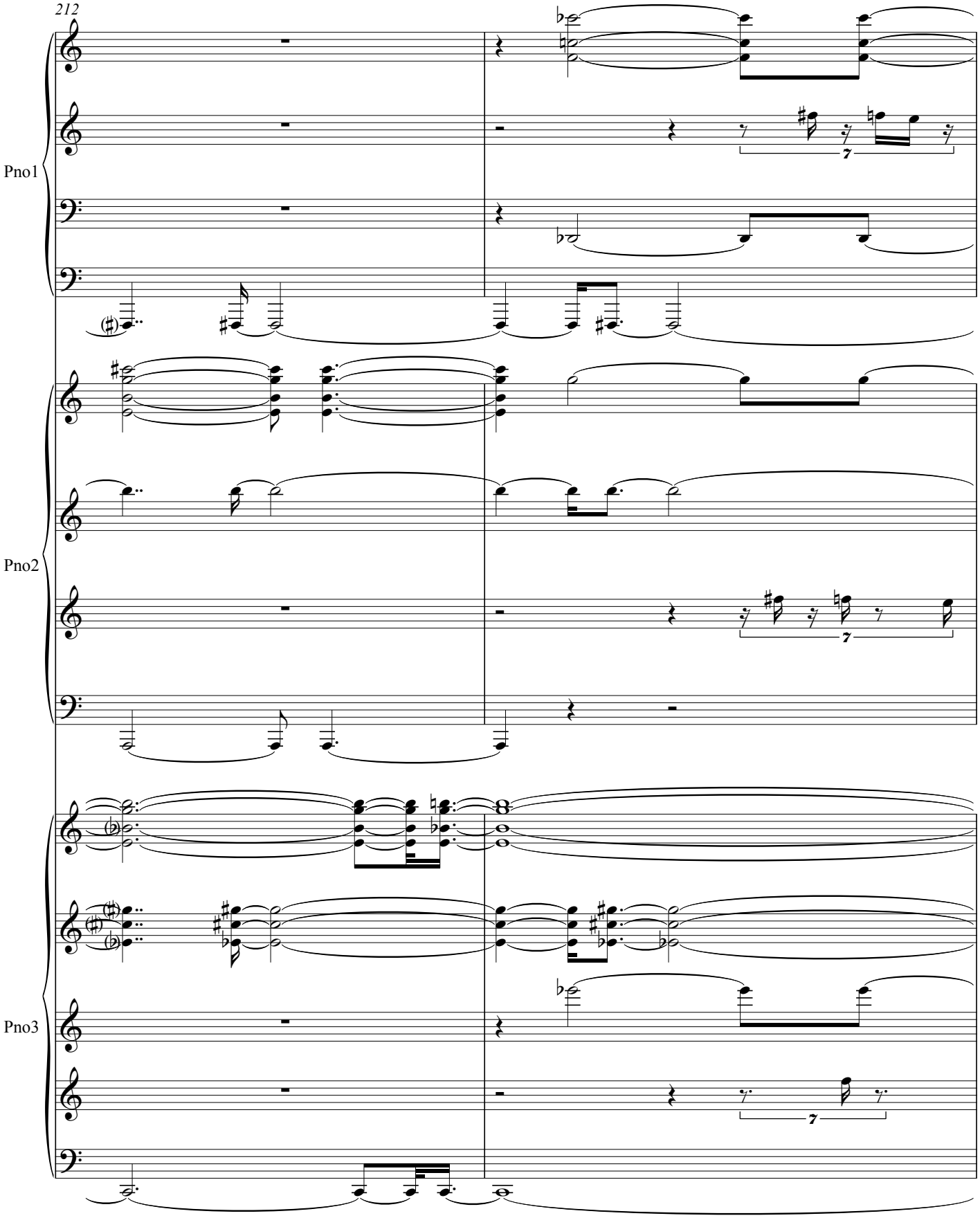
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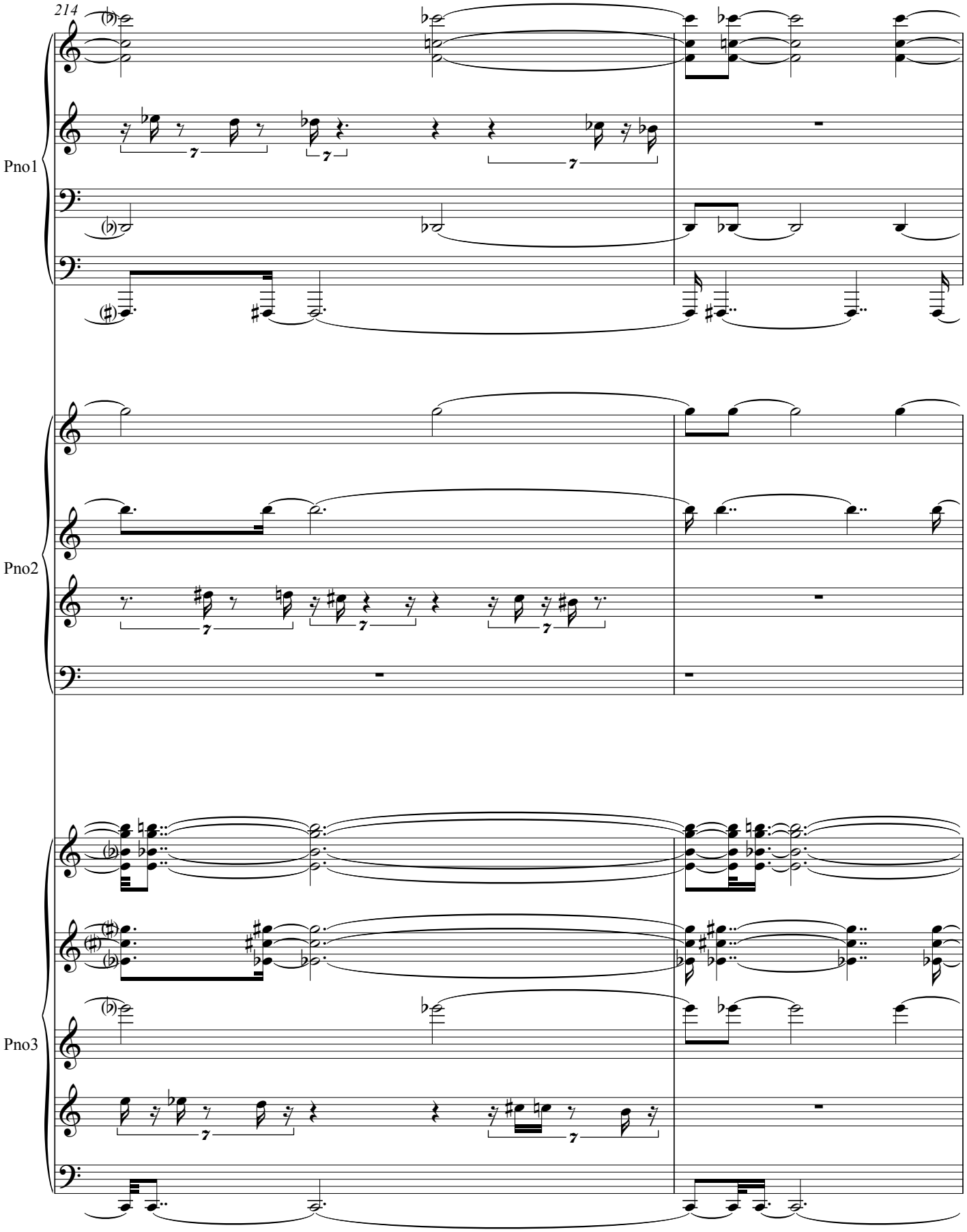
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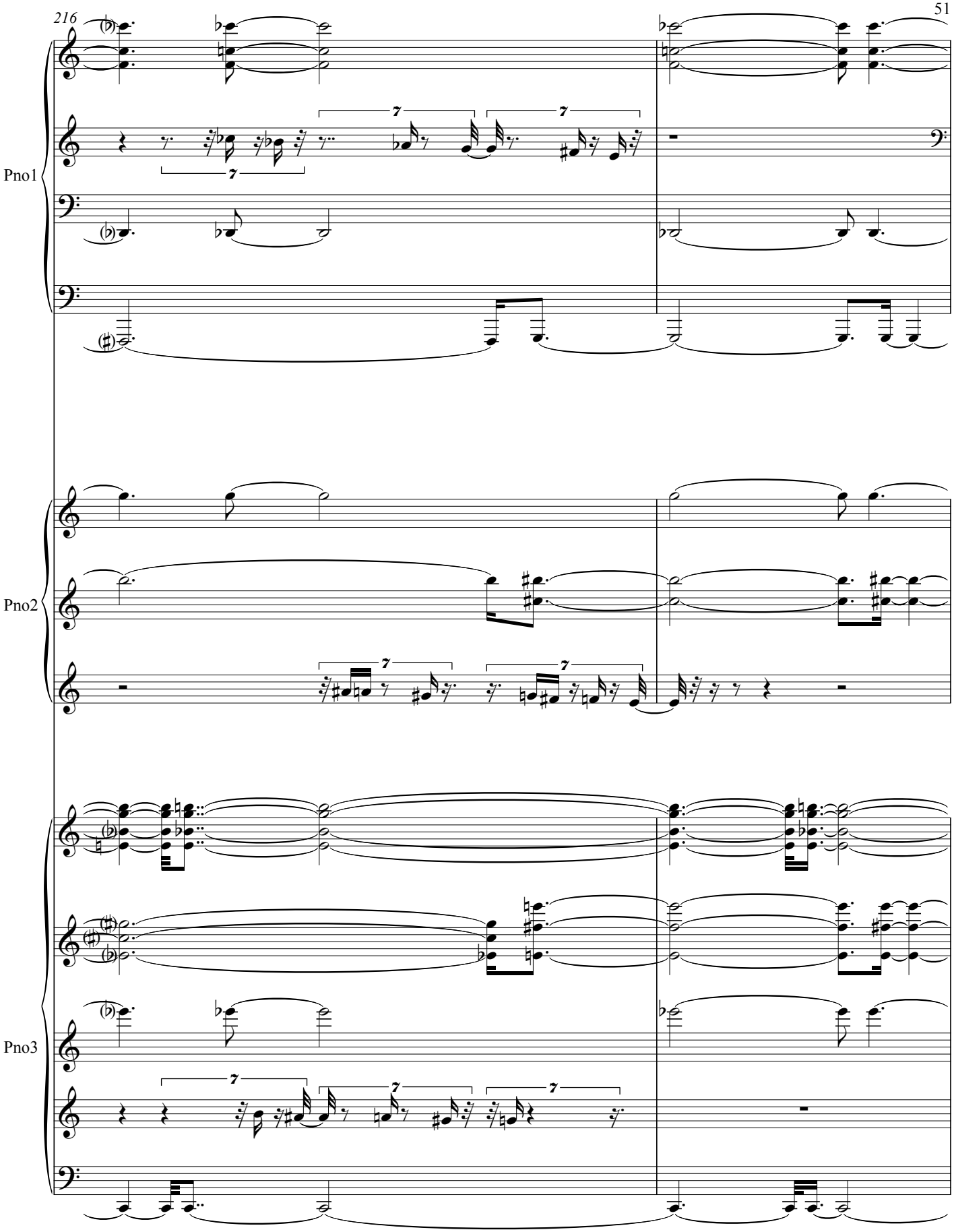
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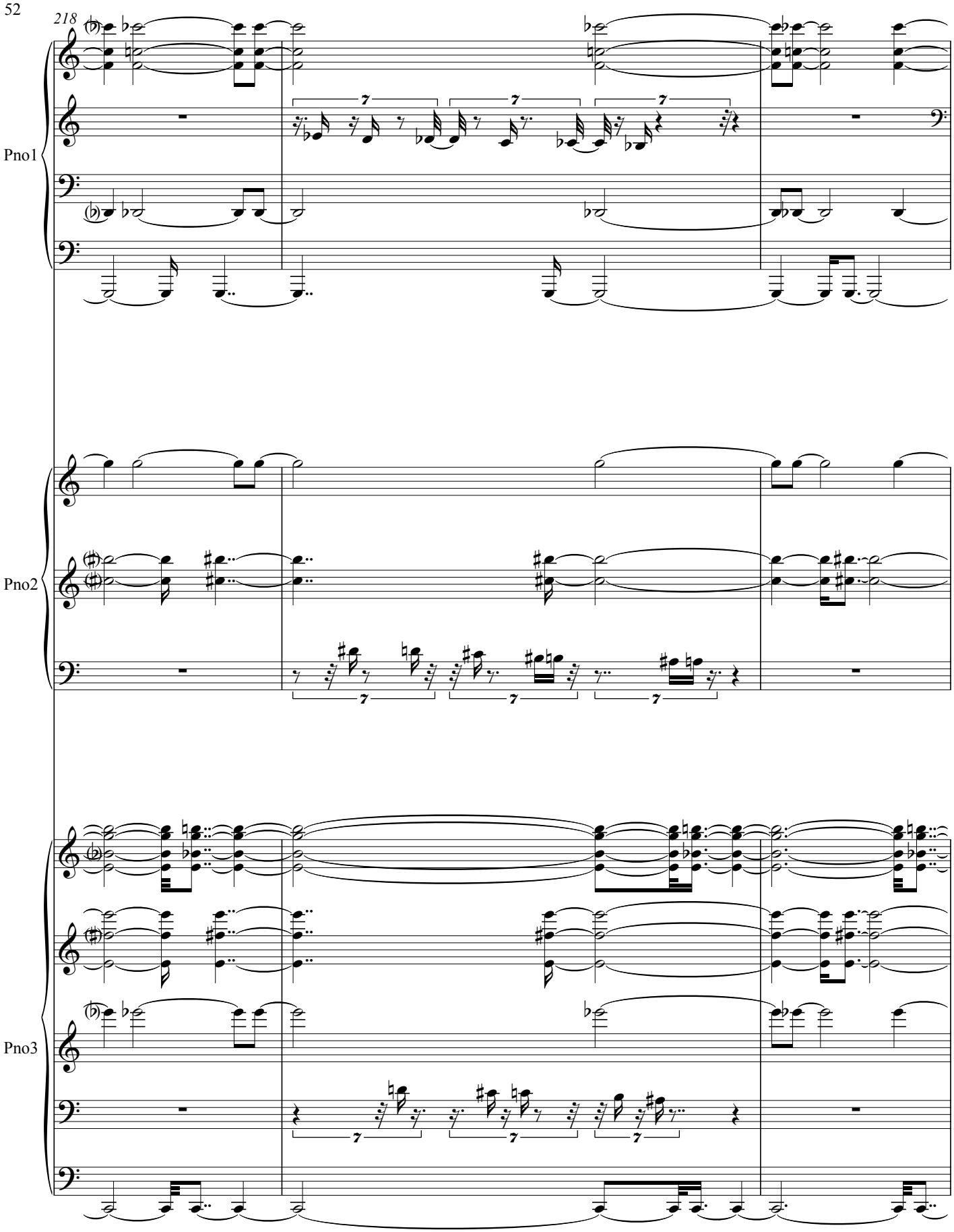
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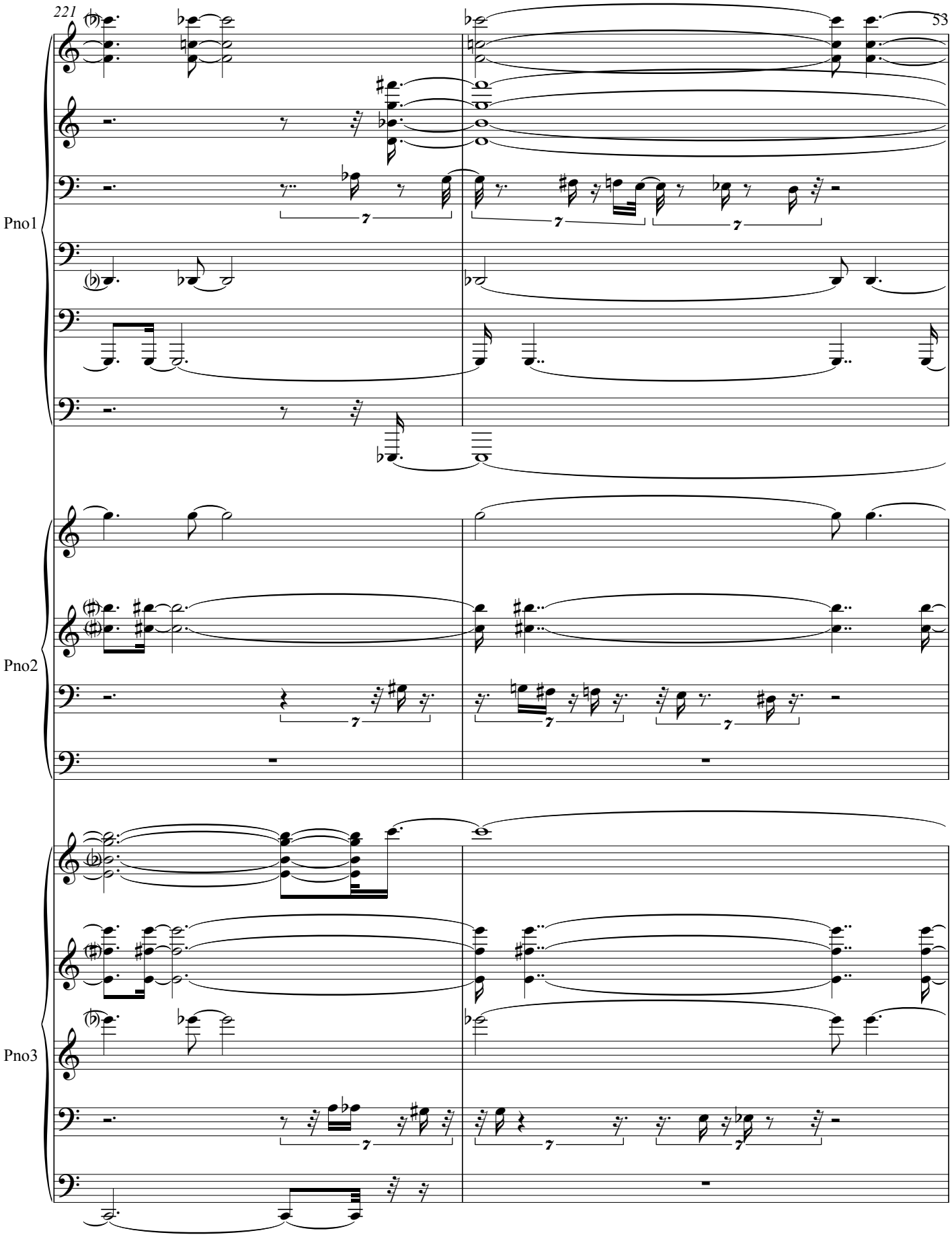
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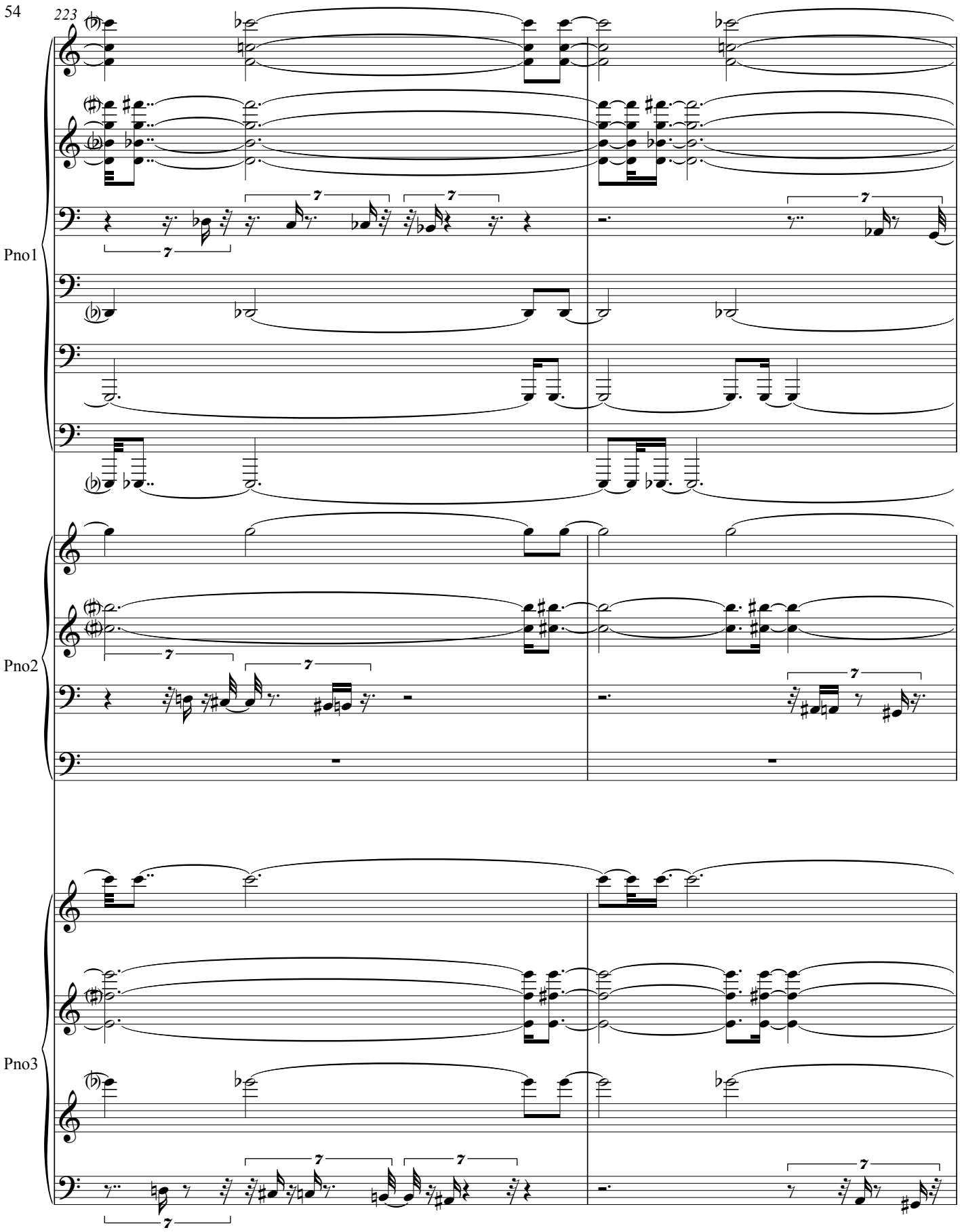
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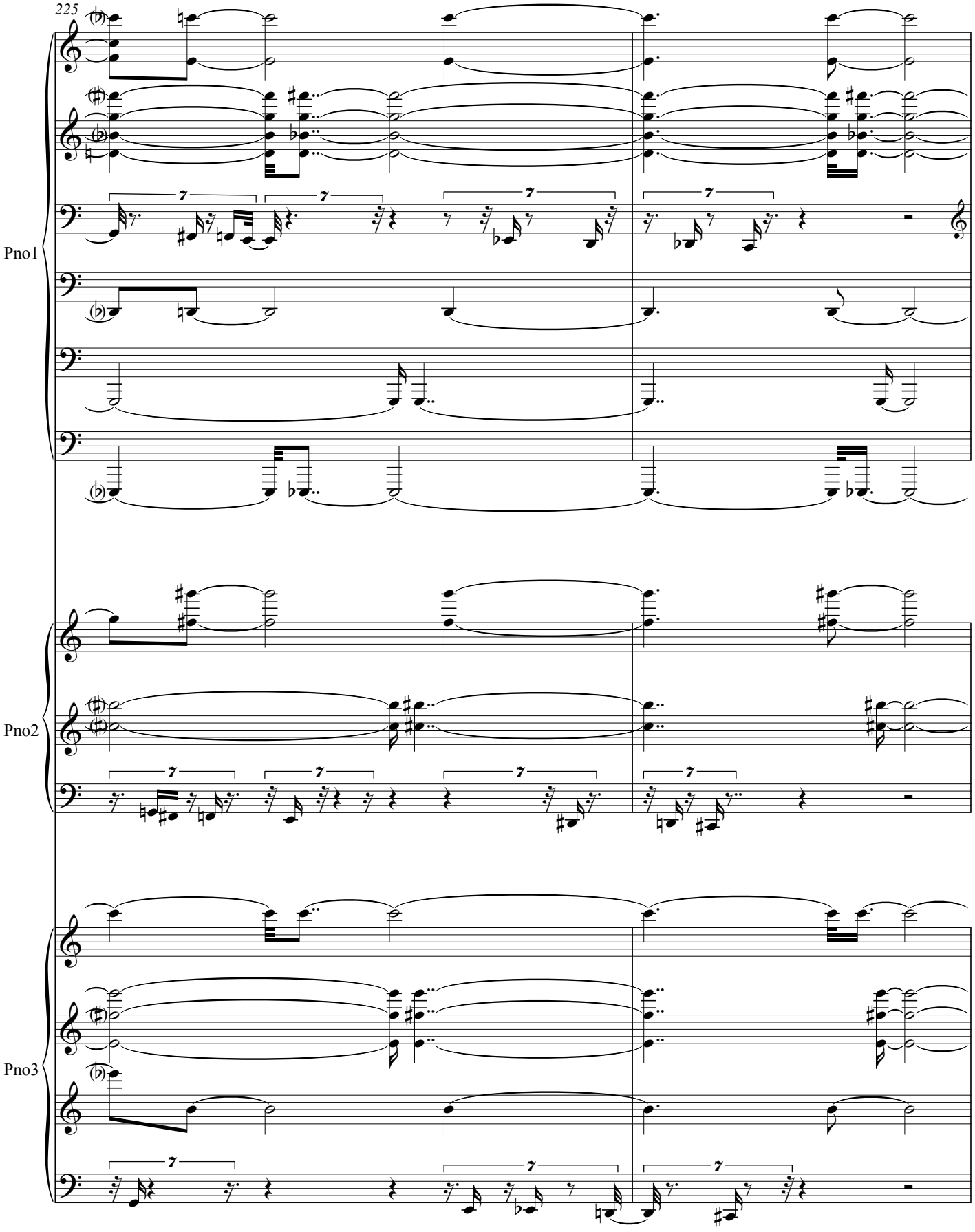
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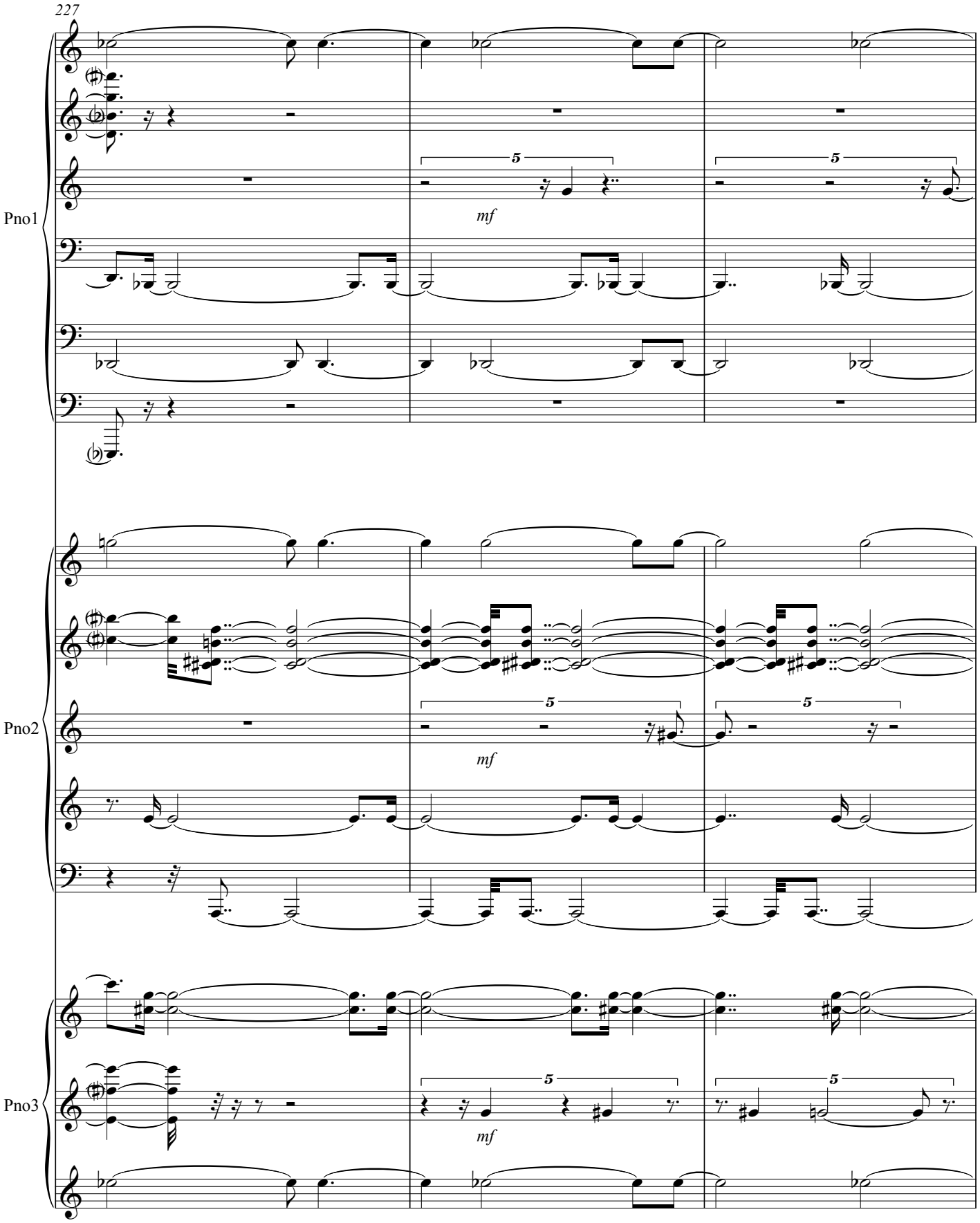
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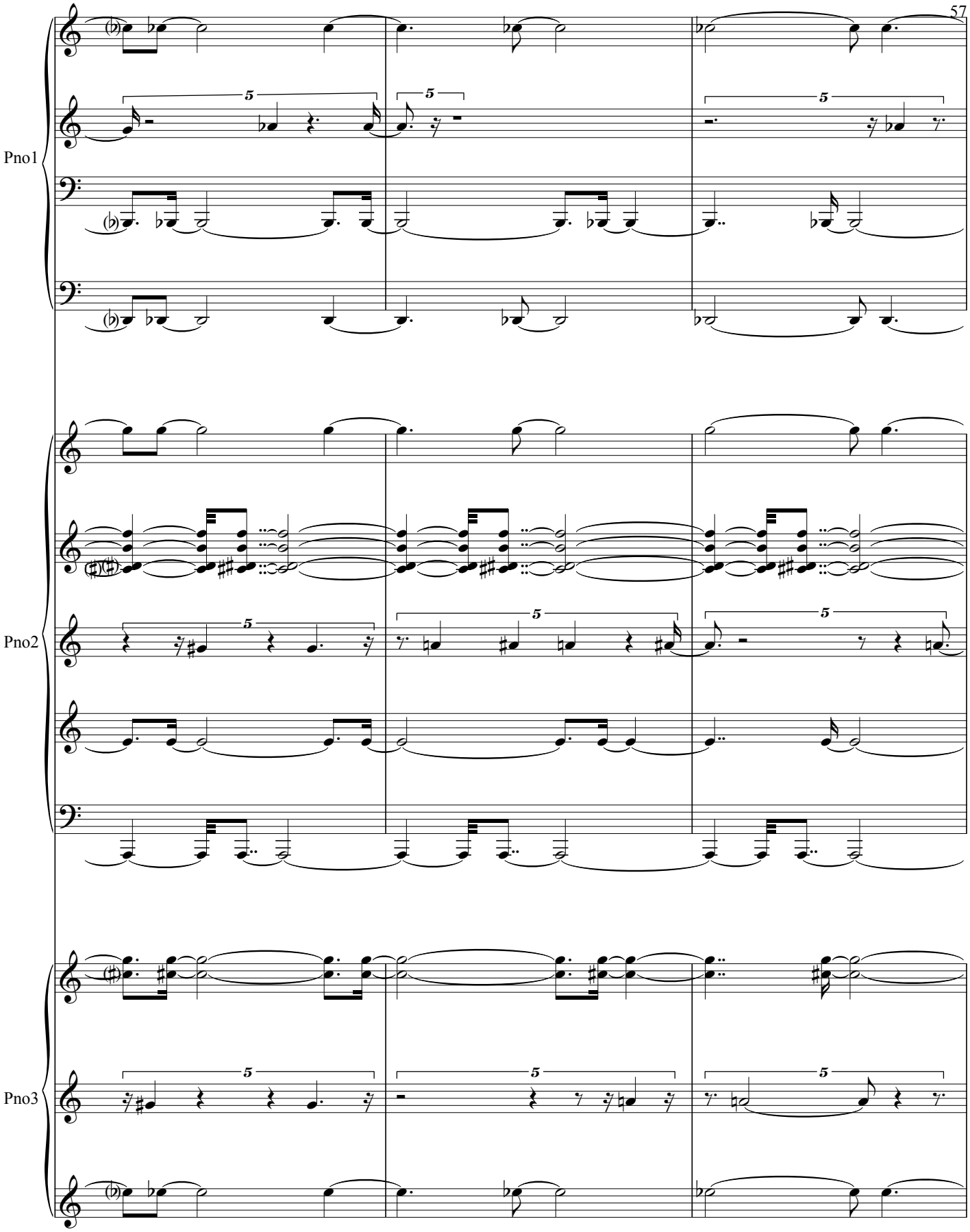
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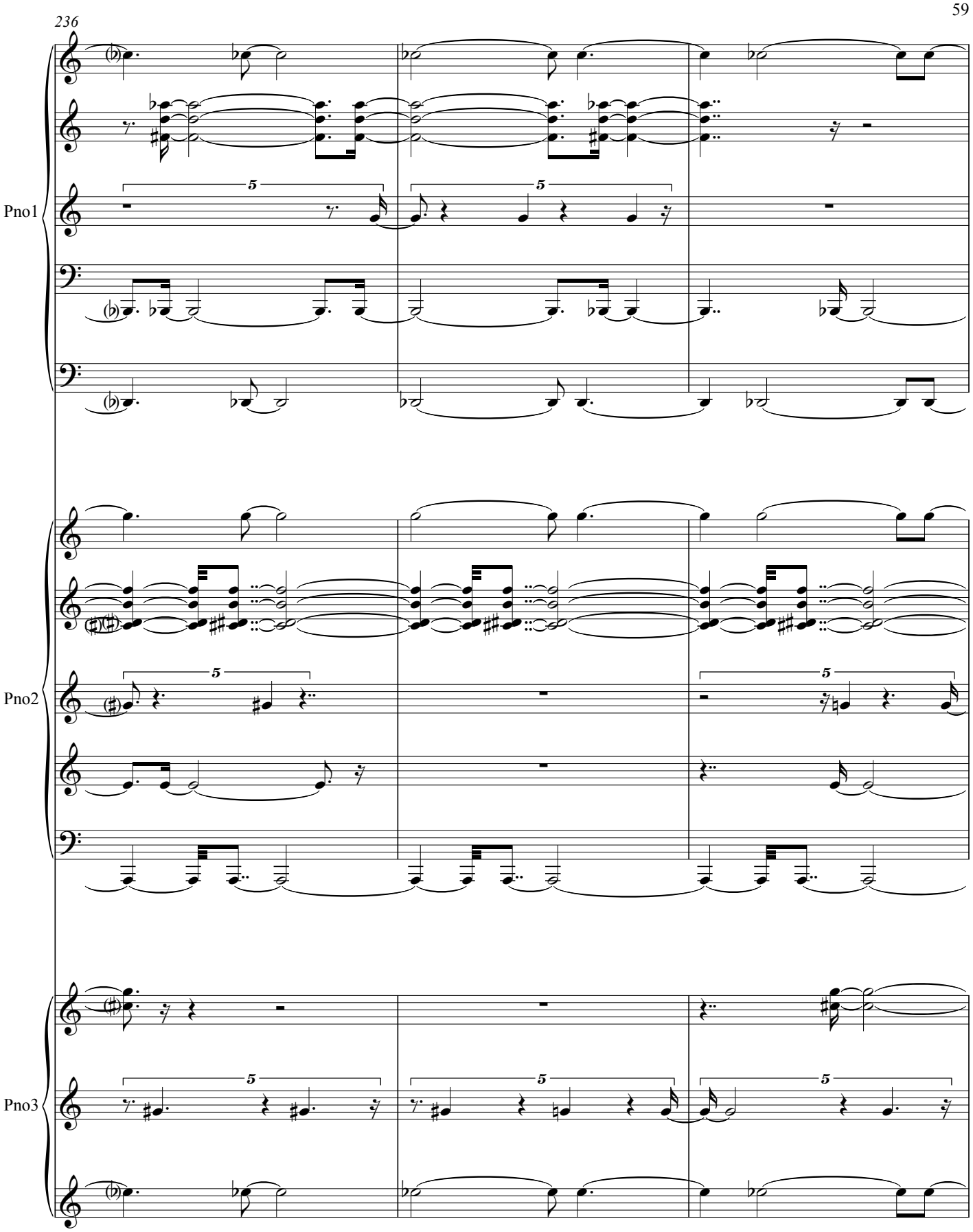
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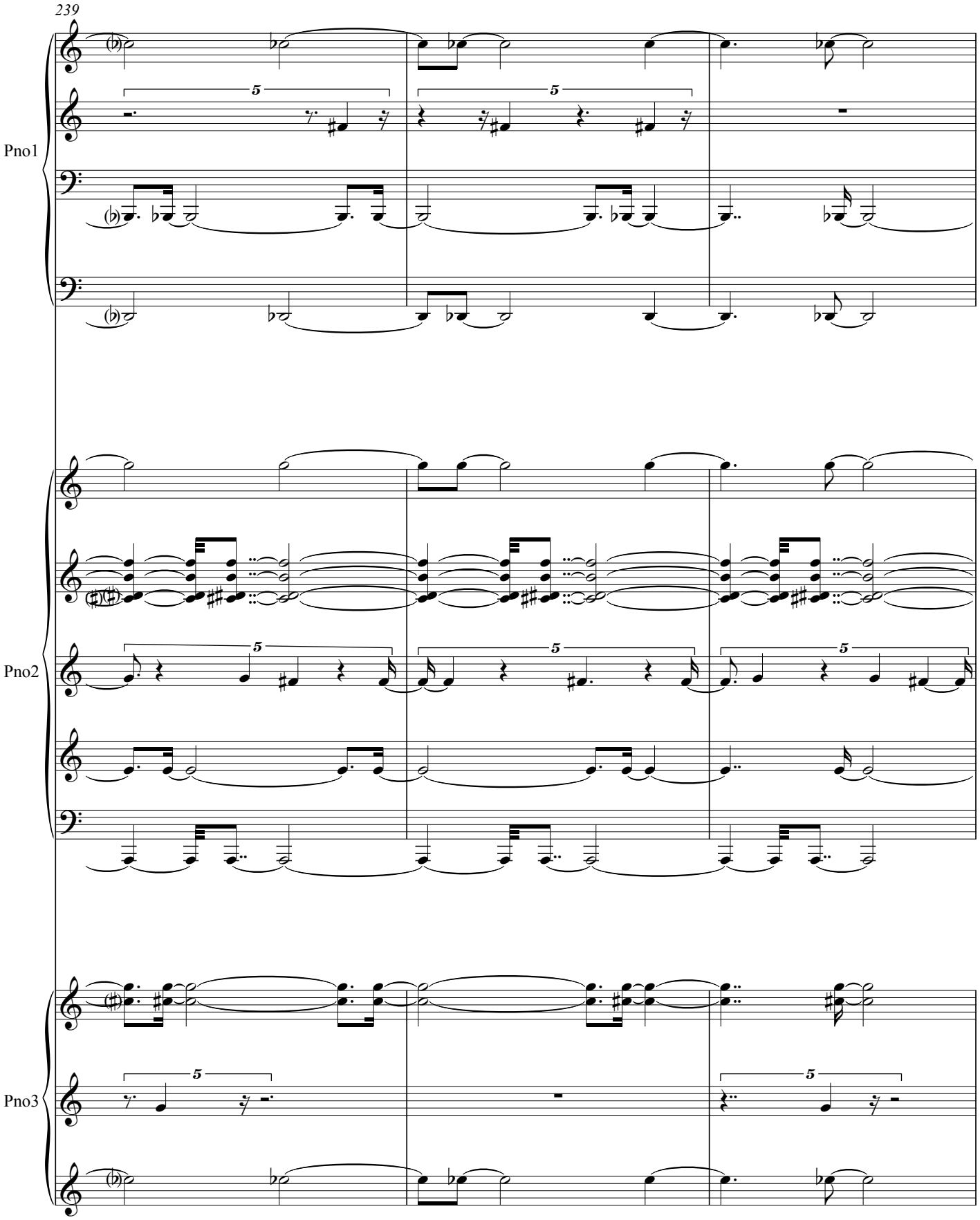
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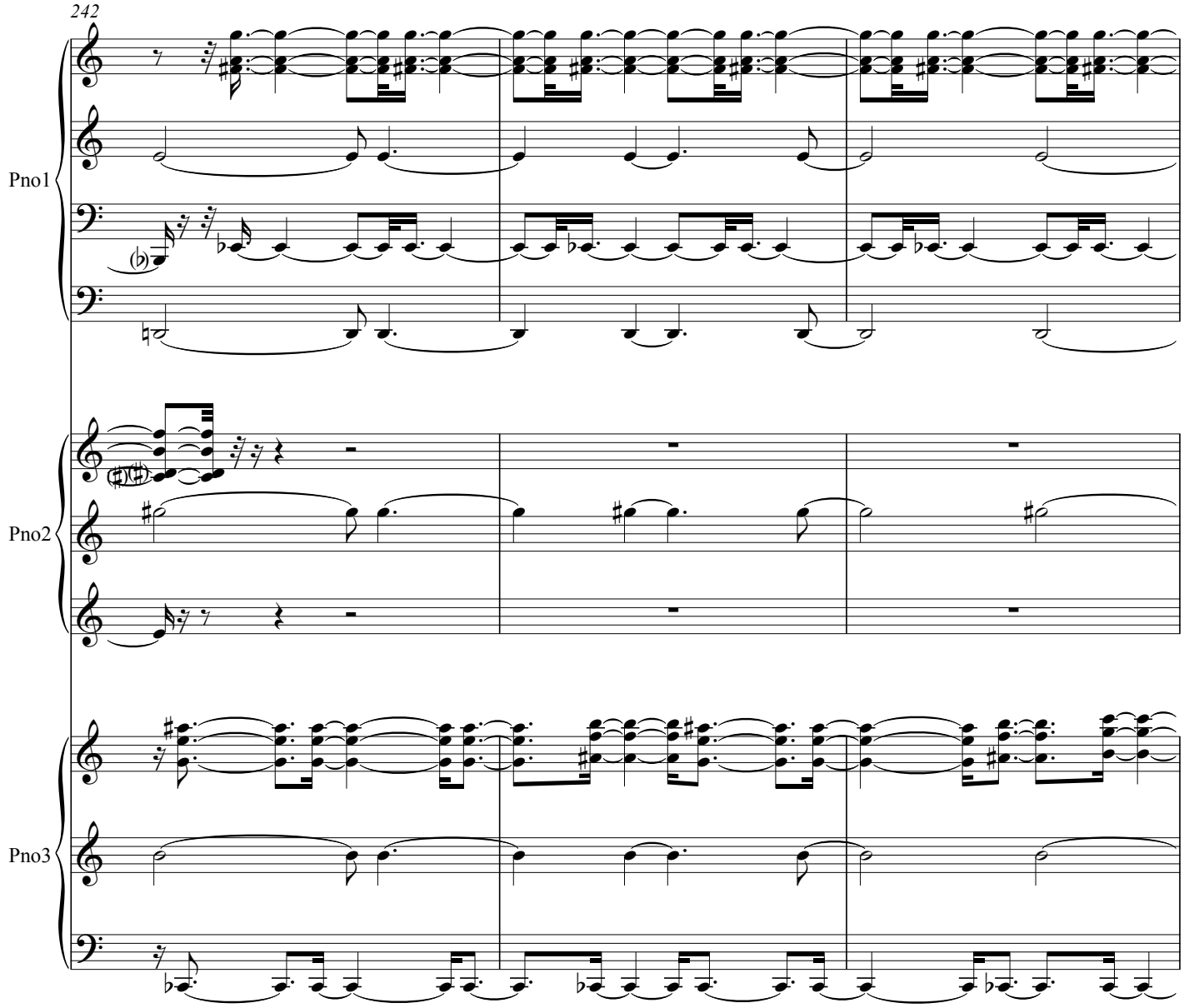
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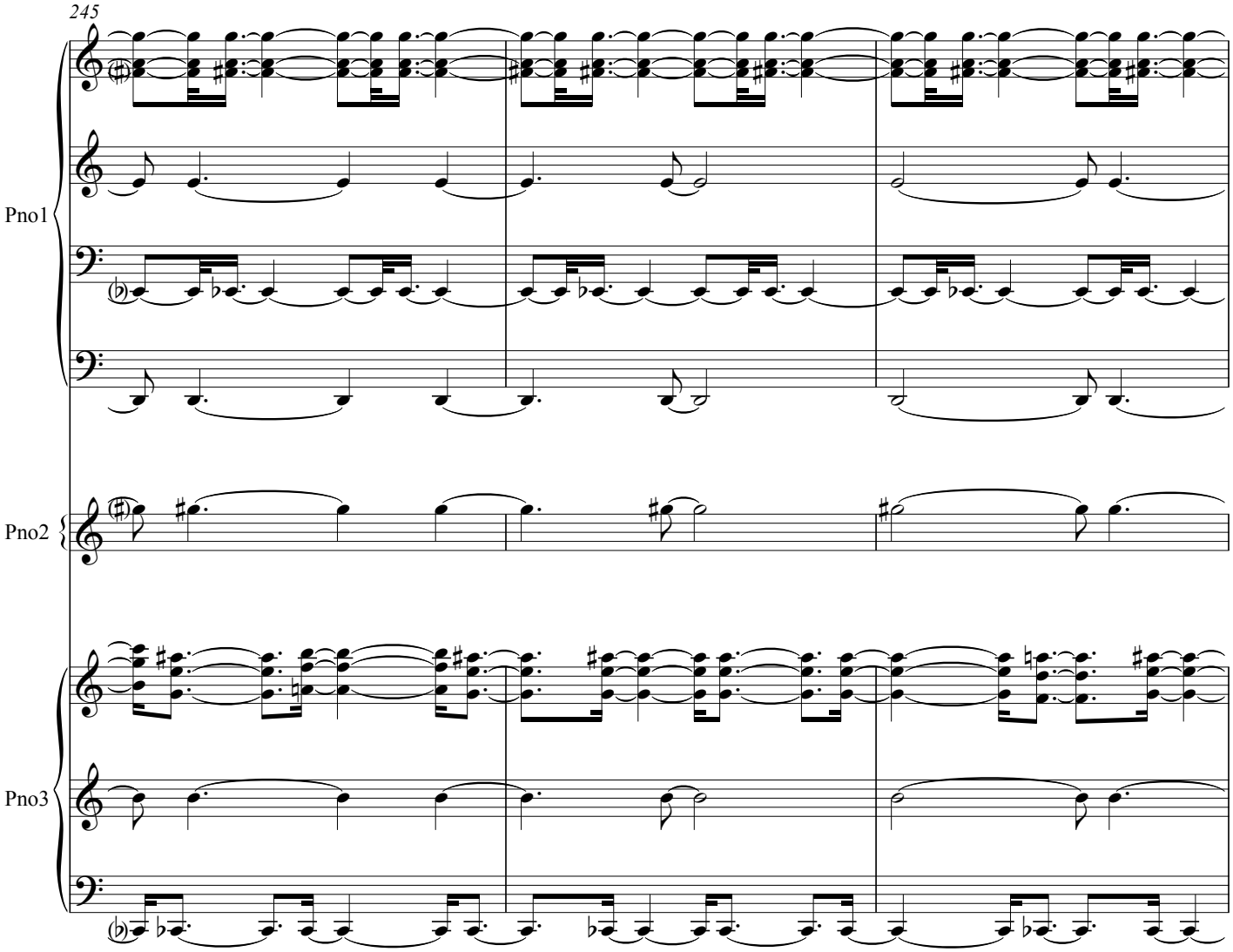
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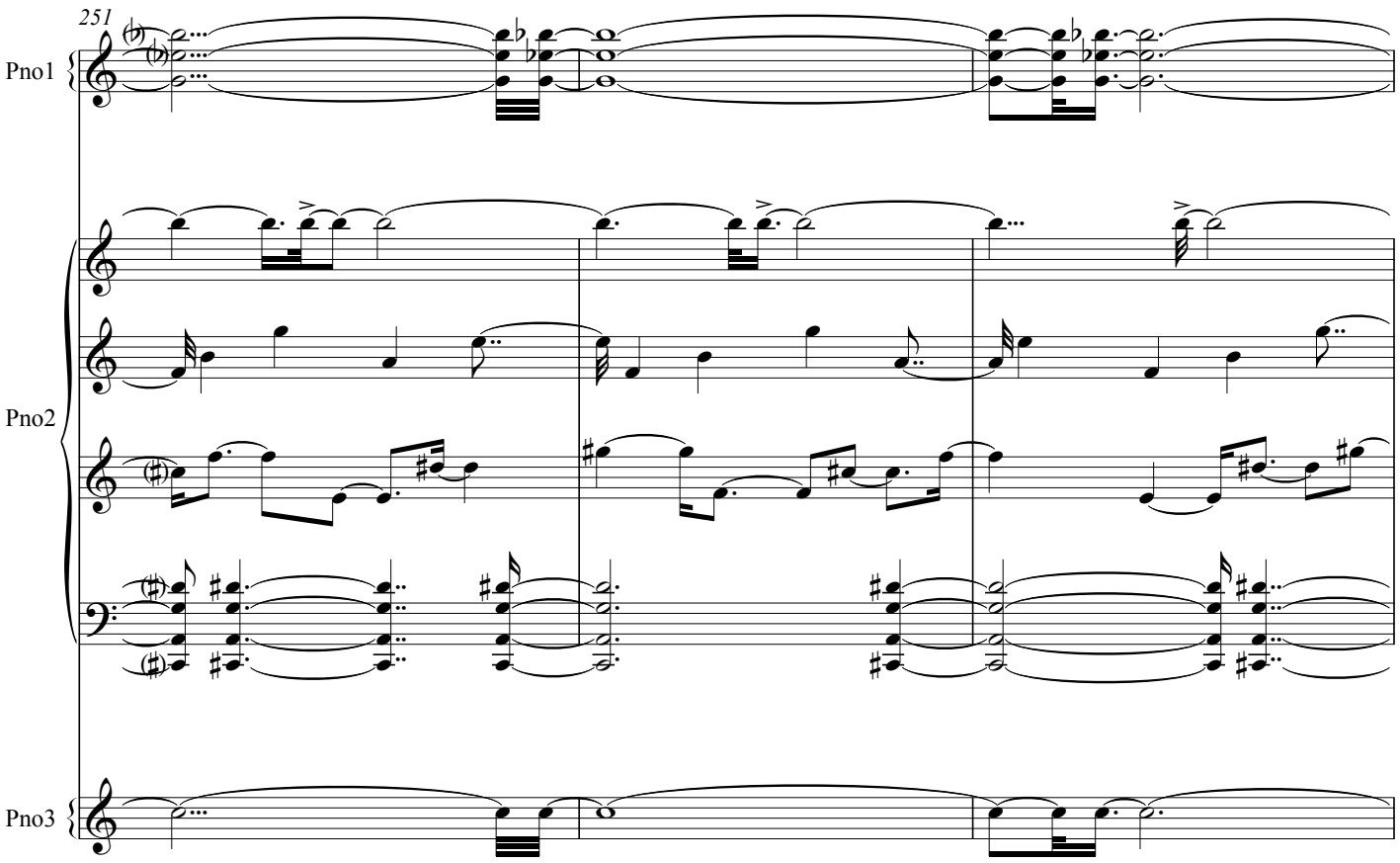
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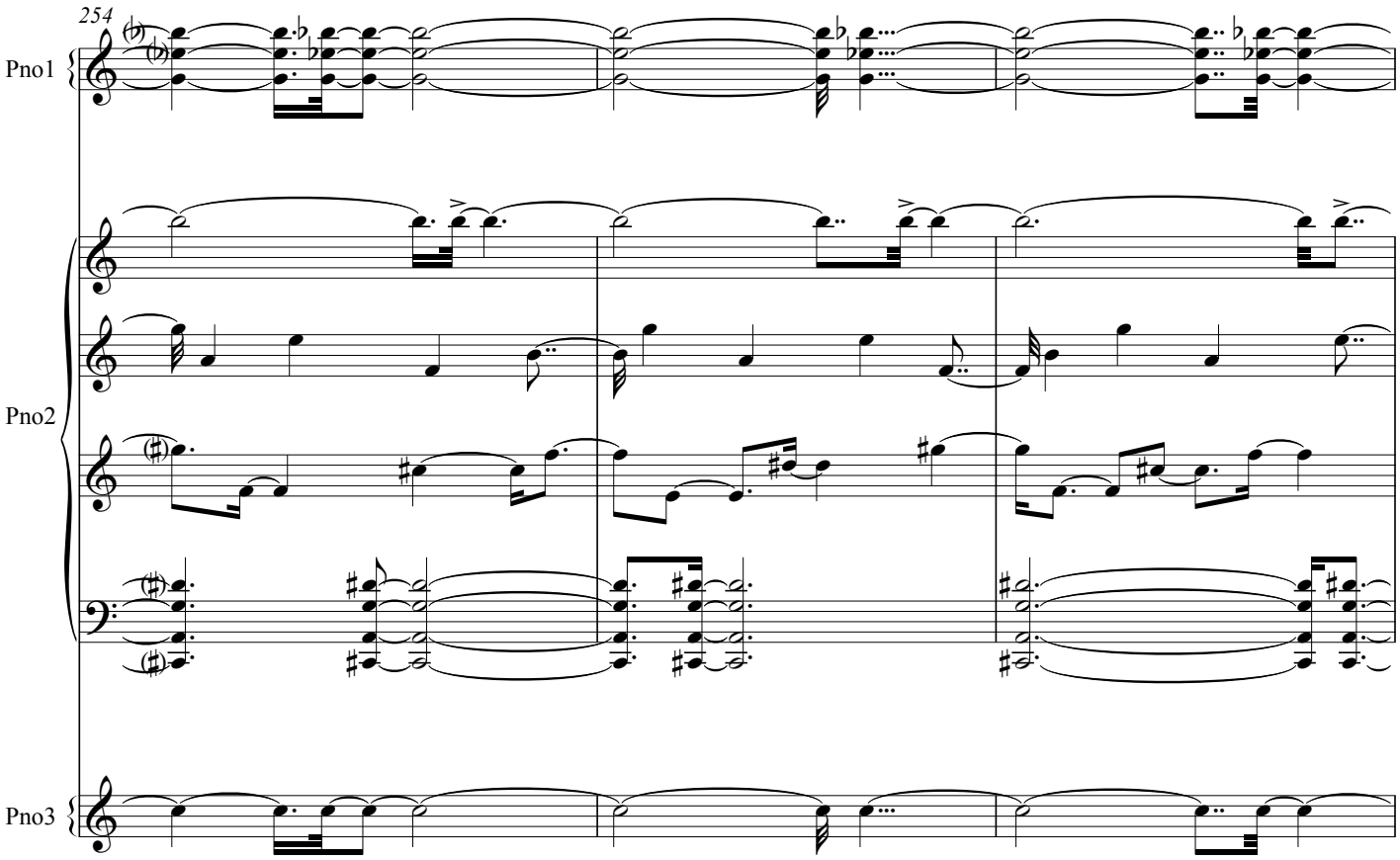
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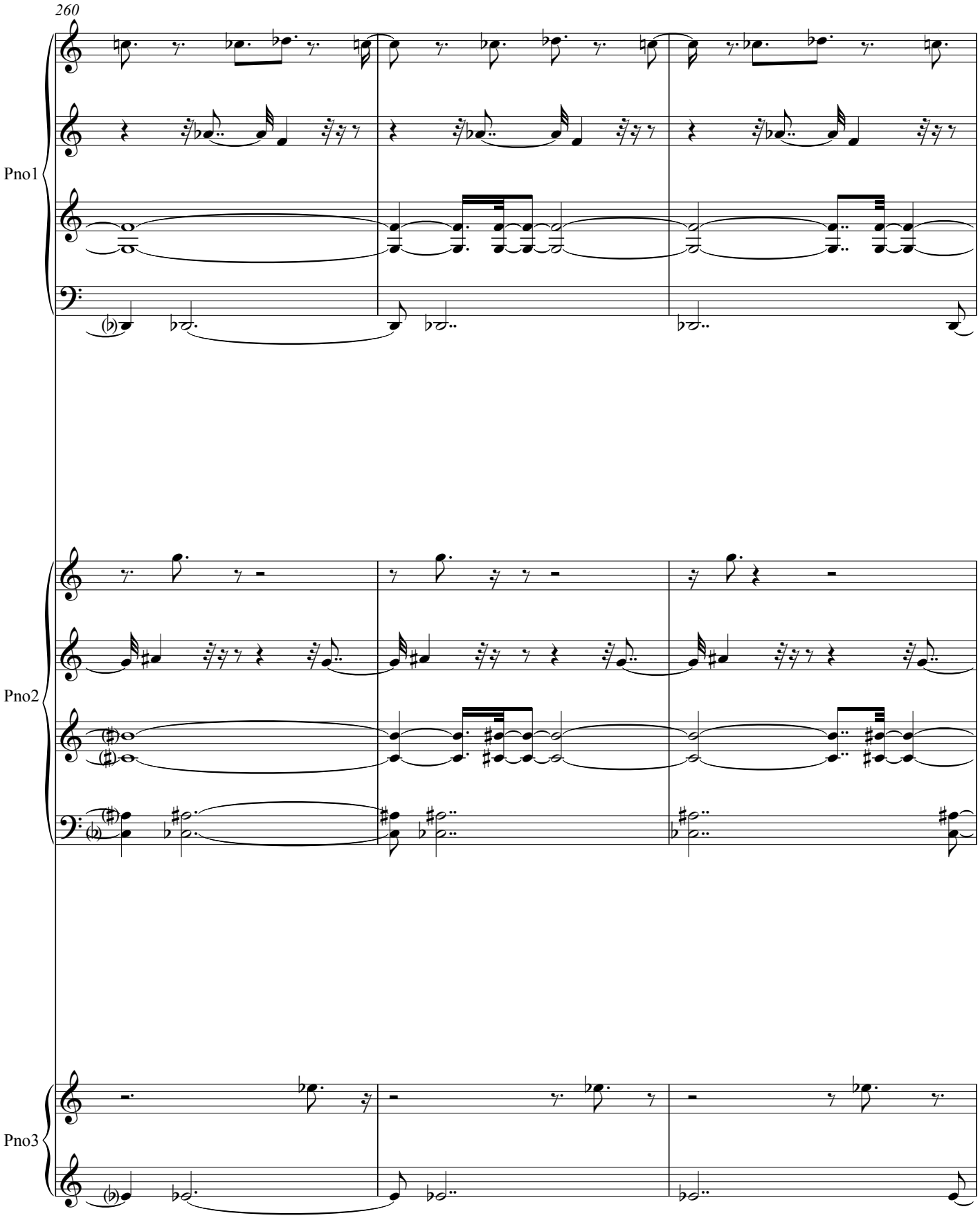
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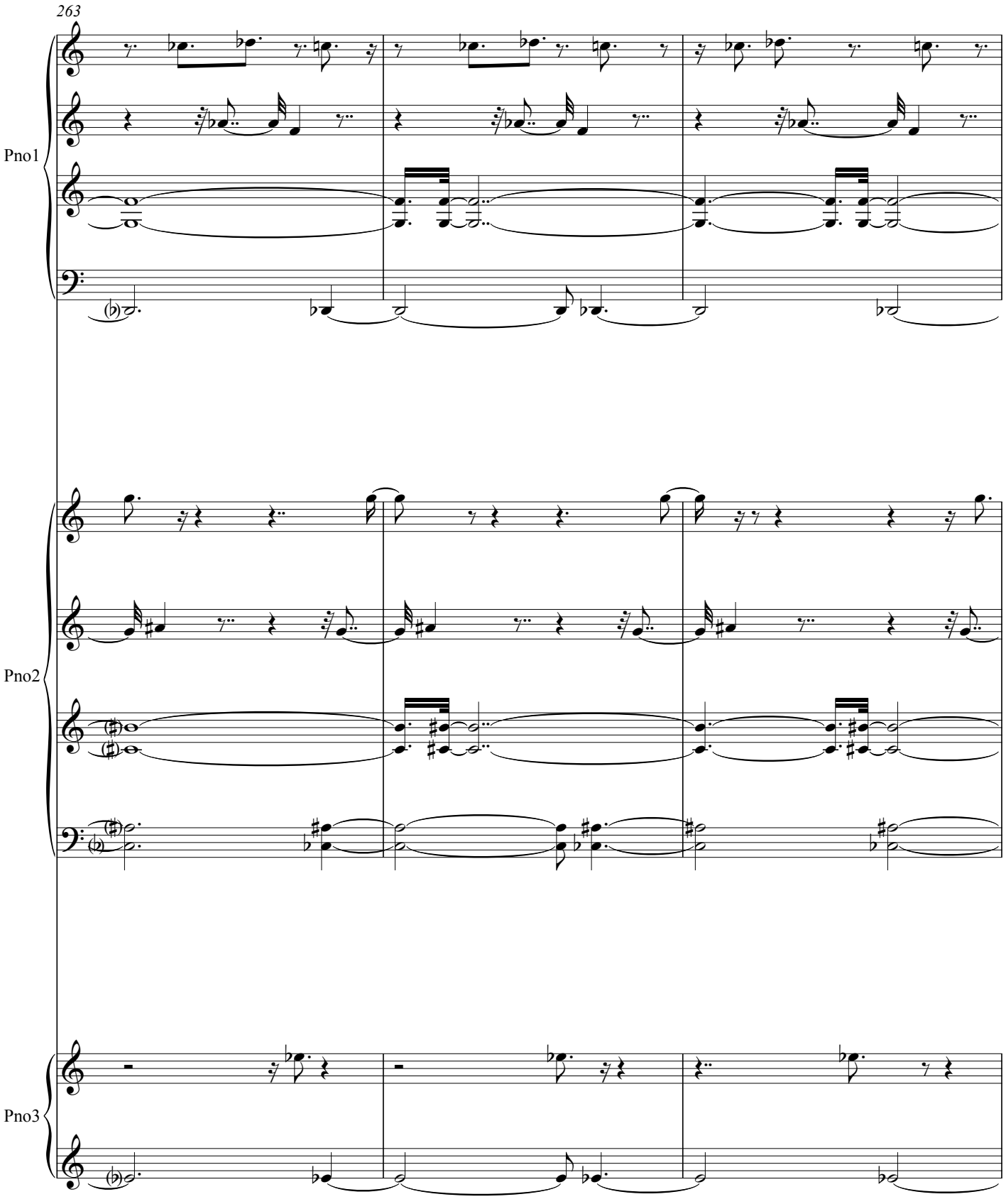
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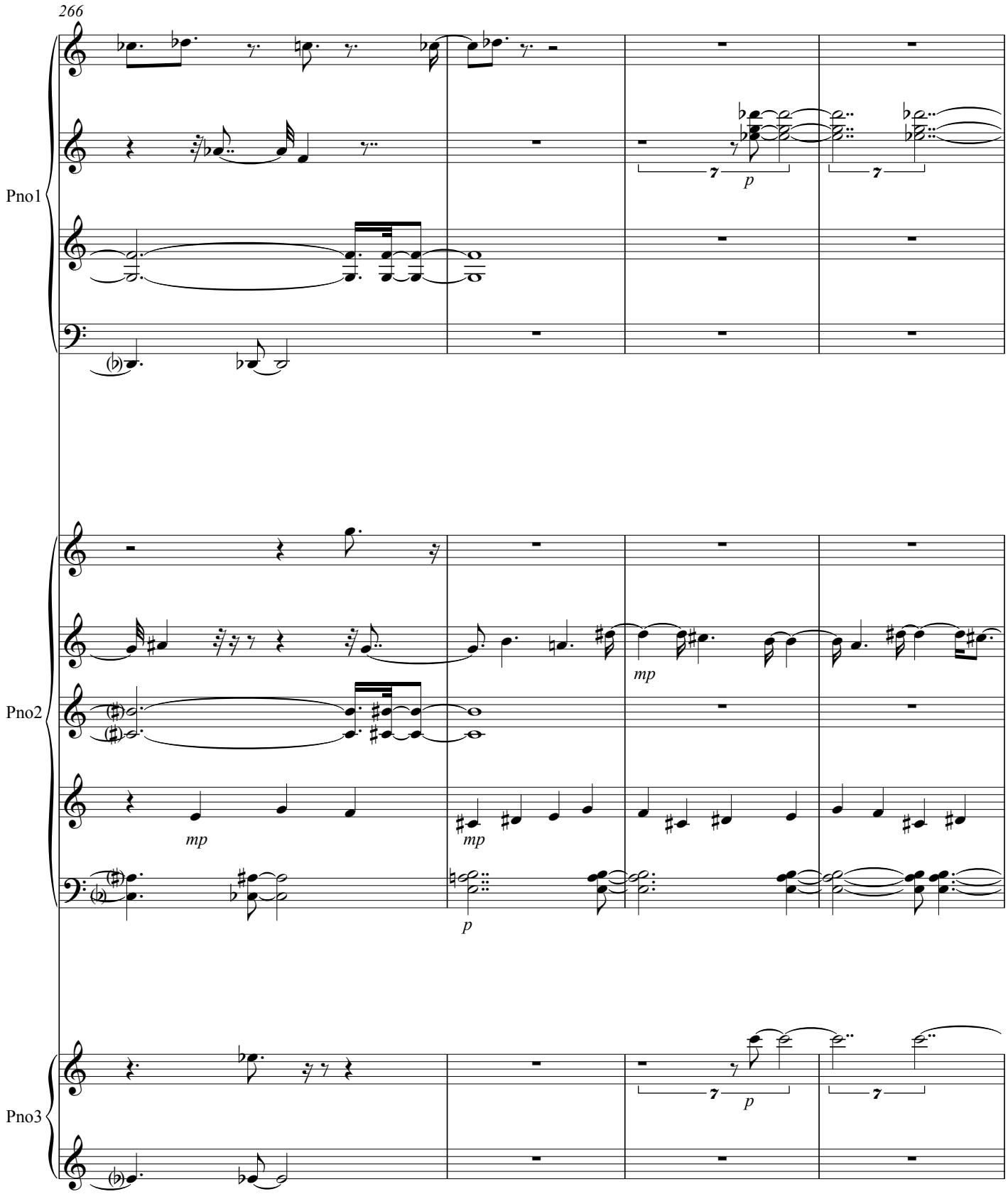
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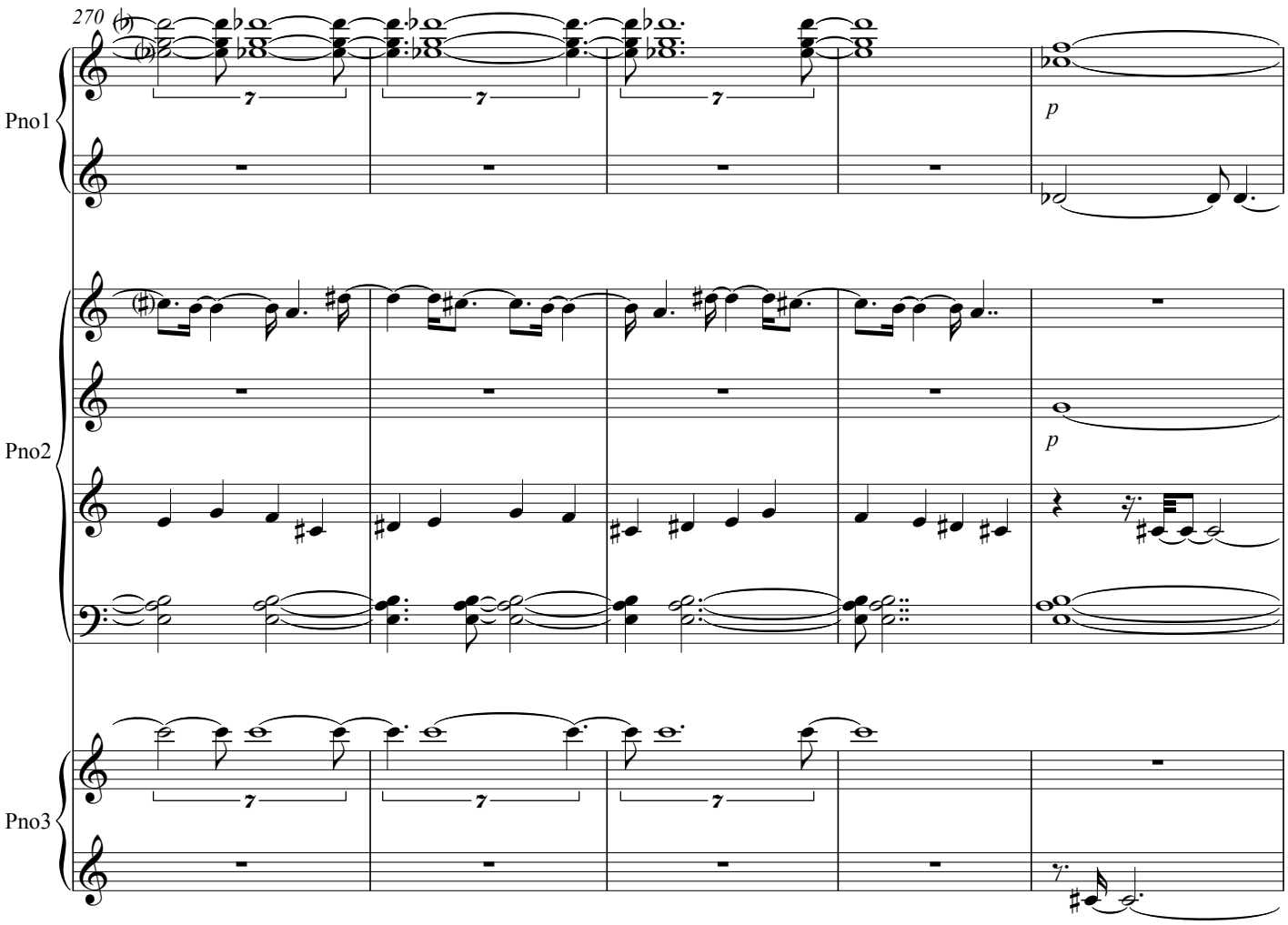
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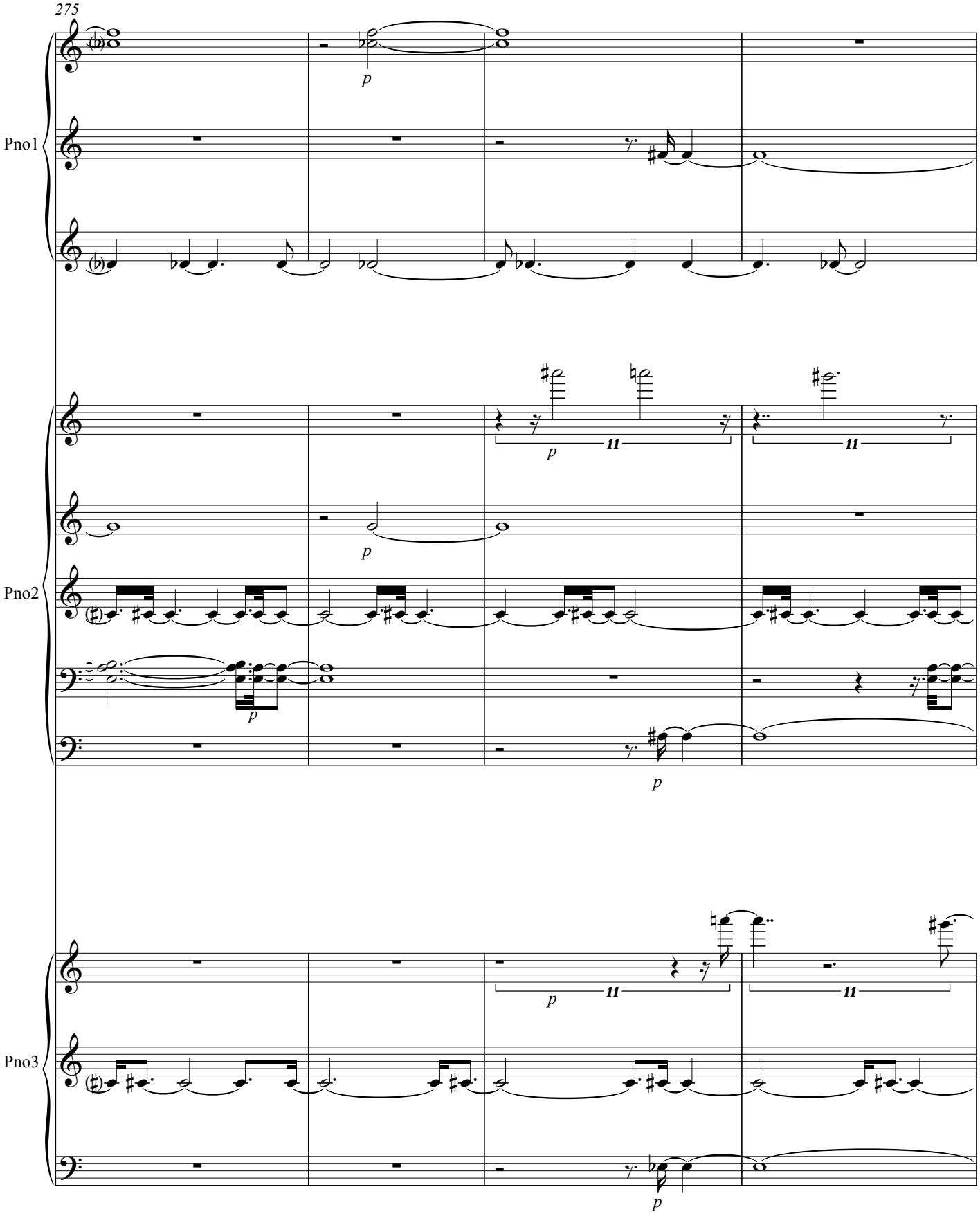
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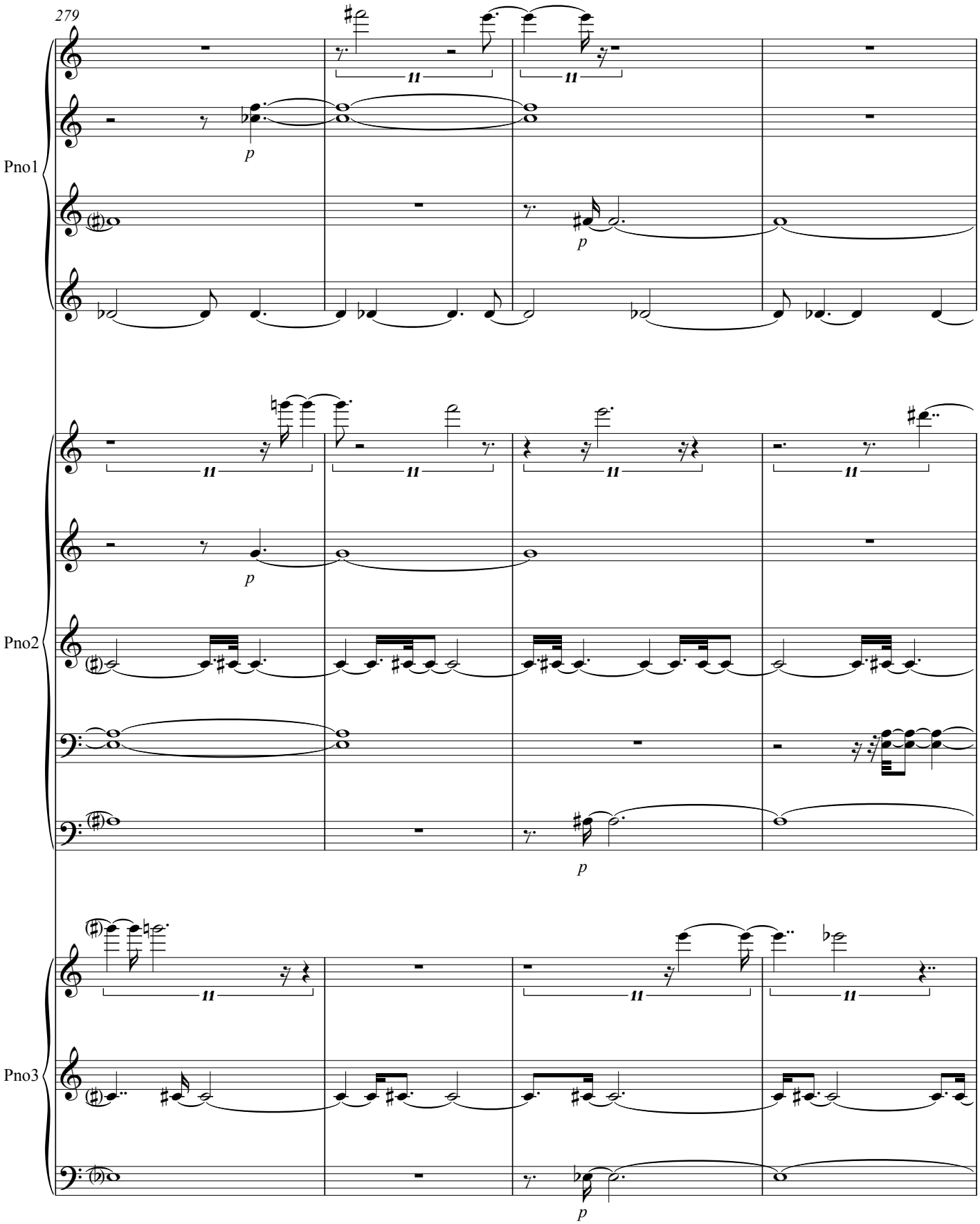
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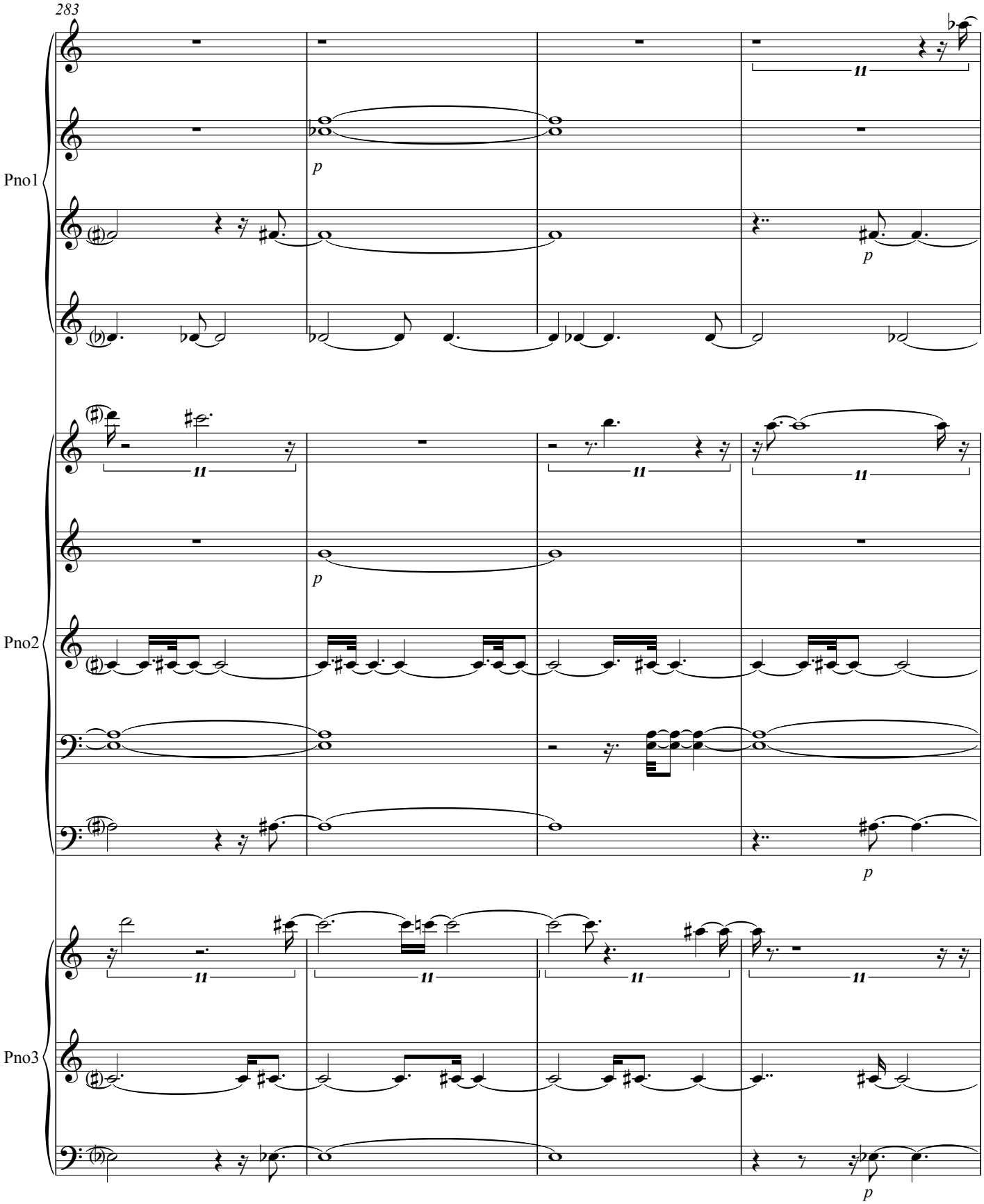
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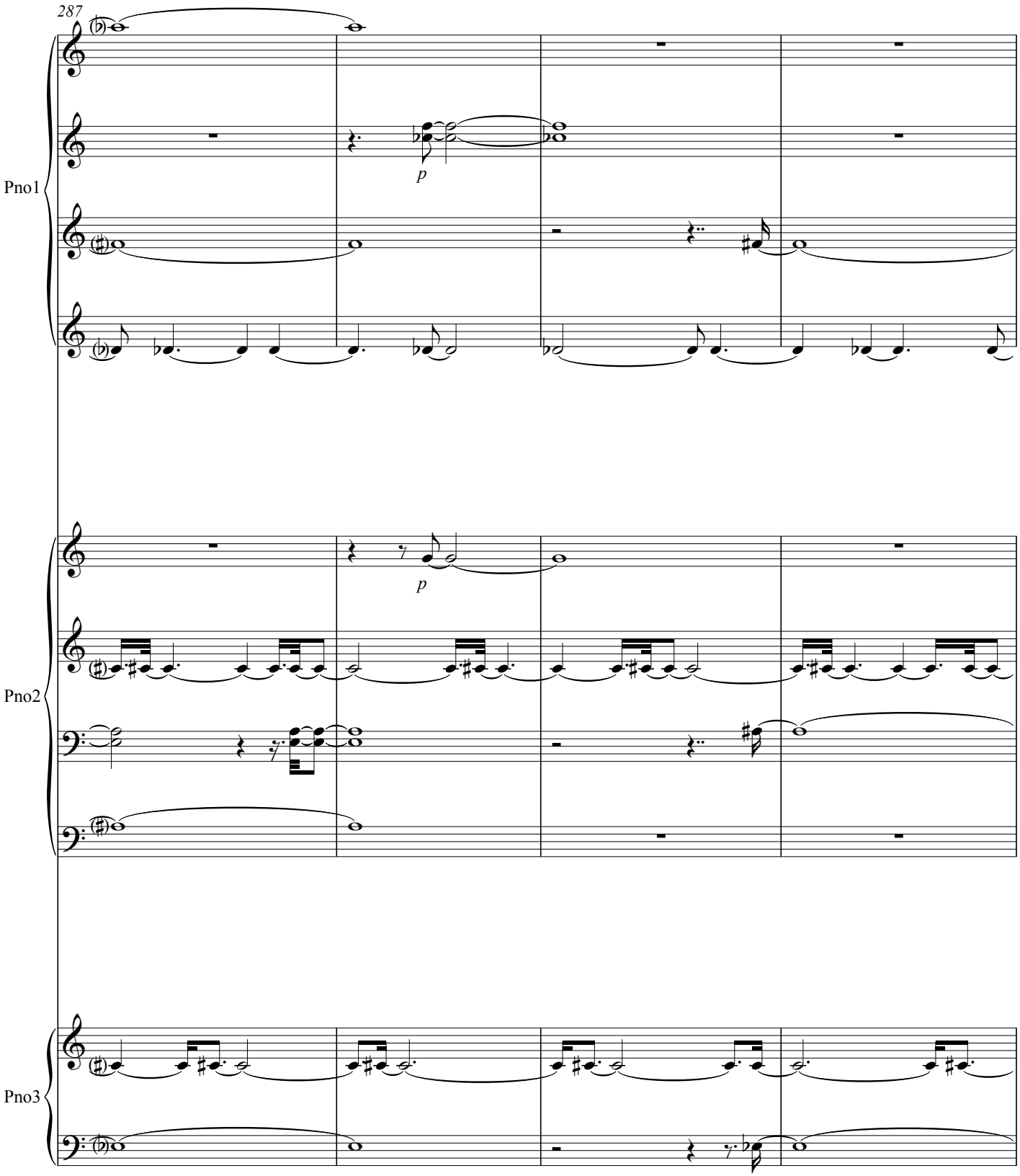
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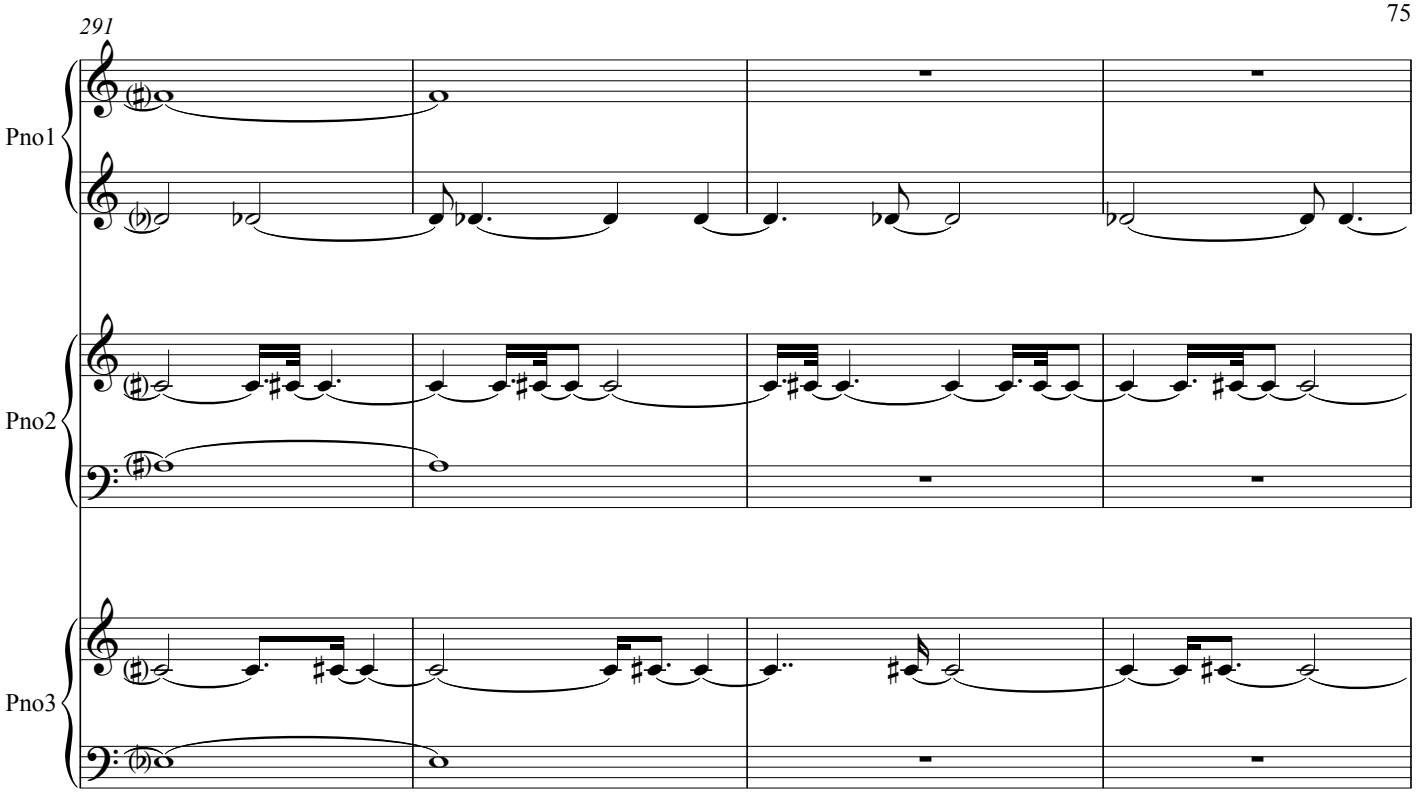
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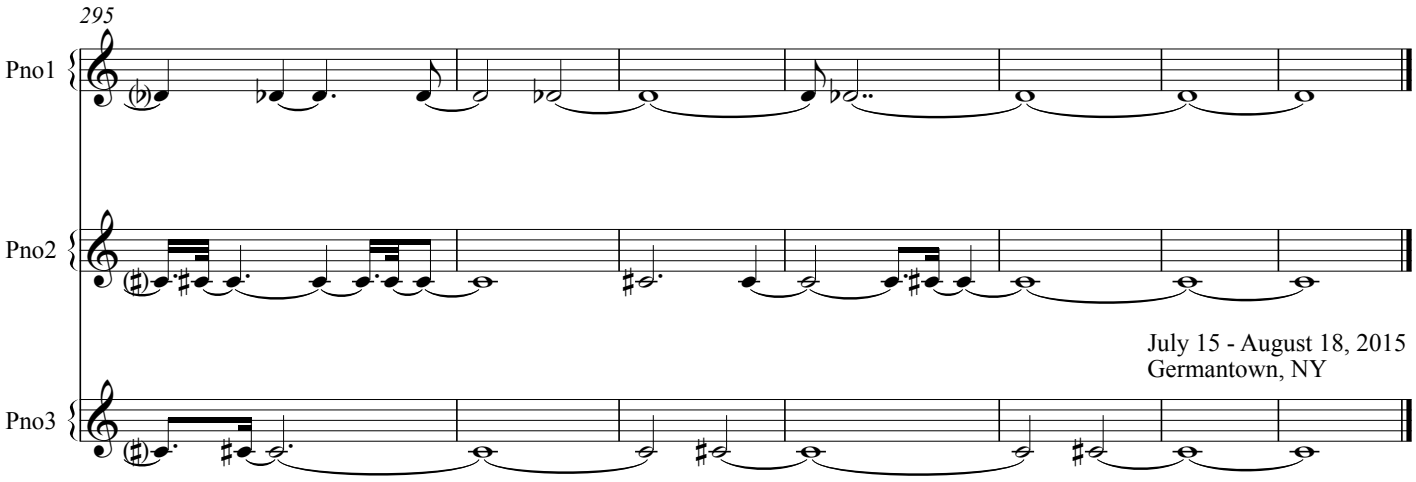
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July 15 - August 18, 2015
Germantown, NY